Stochastic Optimization System for Bank Reverse Stress Testing

GIUSEPPE MONTESI
School of Economics and Management, University of Siena, Italy

GIOVANNI PAPIRO
School of Economics and Management, University of Siena, Italy

MASSIMILIANO FAZZINI
Valuecube, Siena, Italy

ALESSANDRO RONGA
Valuecube, Siena, Italy

- We present a **reverse stress test methodology based on a stochastic simulation optimization system**.

- **Reverse stress test analysis is aimed at finding a solution to an inverse problem**: detecting the scenarios on the edge between the condition of viability and default, that is the exact conditions in a small set of risk drivers that trigger the bank's default, which from a regulatory point of view can be identified by the breaking of a minimum regulatory capital threshold, such as TSCR.

- This methodology enables users to **derive the set of assumptions of key risk drivers that**, by triggering a preset key capital indicator threshold, **causes the bank's default**, defines the **reverse stress test scenario**.

- The article provides a **theoretical presentation of the approach** and an **example of application** of the proposed methodology **to the Italian banking sector**.

- We also show how to take into account some relevant risk factor interactions and second round effects, such as **liquidity-solvency interlinkage** and modelling of **Pillar 2 risks** including **interest rate risk, sovereign risk and reputational risk**.

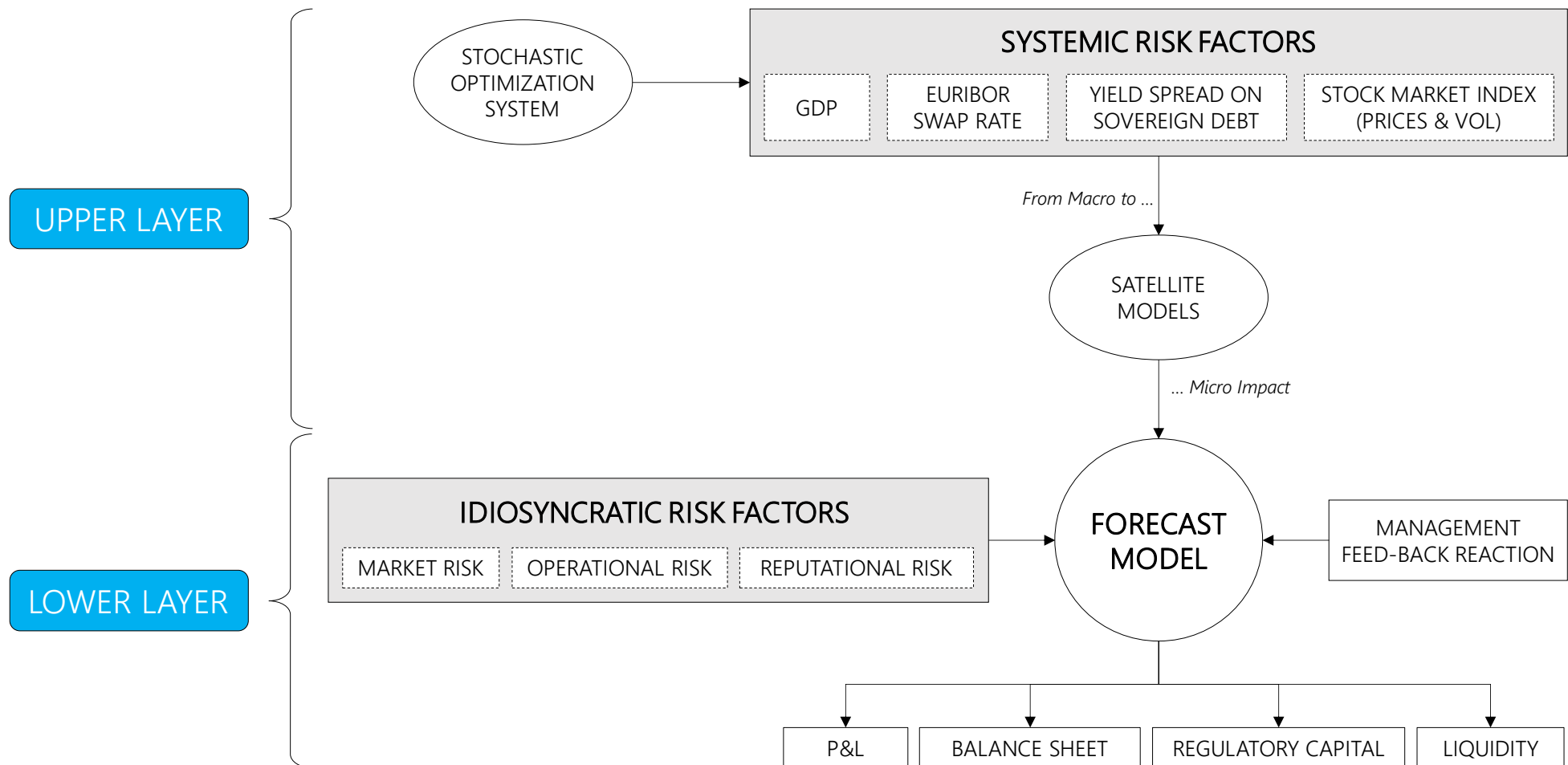## Reverse Breaking Points Edge & Probability of Breach

- Reverse Stress Testing is a **complex problem that has multiple solutions**, since there are many combinations of risk factors through which a bank's may breach the relevant threshold of its key risk indicator.

- Assessing the probability of breach is an easier task since we just have to detect all the scenarios which can cause the key risk indicator to fall below the relevant threshold in the breach area; whereas **in reverse stress testing we need to identify only those solutions which just trigger the threshold** and lay on the edge between the condition of viability and default.
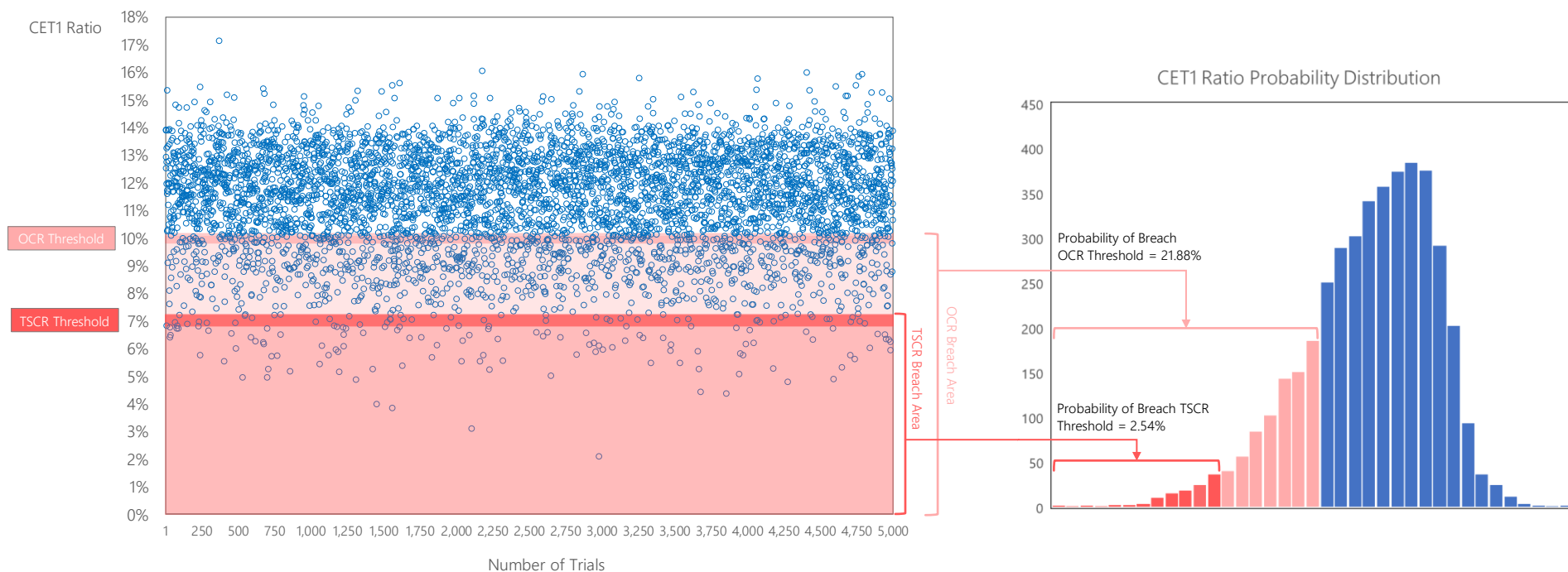
Reverse Stress Testing Involves Two Types of Problems

- A **computational issue related to the derivation of the reverse solutions,** which can be resolved through an efficient quantitative technique to find out all those combinations of risk factors that can trigger the threshold (the reverse breaking points).

- The **choice of a criterion to select the reverse stress test scenario** from among all the solutions obtained that we can consider as the reverse stress test scenario (this issue cannot be addressed in purely quantitative terms and requires ultimately subjective decisional criteria).

- **Systemic risks** are modeled through **stochastic variables** and are **subject to the optimization process**, their number and nature can be freely chosen, but the higher the number of explanatory variables and the higher the number of potential solutions.

- **Idiosyncratic risks** are modeled through **stochastic variables** and **not subject to the optimization process**

- Satellite models can be freely adapted and introduced in the modeling framework.

- In simulating conditions of default it is very important to consider in the **model risk factor interactions and second round effects**; we tried to model-in some relevant dynamics among **liquidity, interest rate risk, sovereign risk and reputational risk**.

Country Rating = f(Sovereign Spread)

Bank Rating = f(Country Rating)

$$Bank\ Rating = 1.1808 \cdot Country\ Rating^{0.9107}$$

Deposit Hair Cut Distribution

Operating Cost Increase Distribution

Net Commissions Decrease Distribution

REPUTATIONAL EVENT

CUSTOMERS DROP OUT

DECREASE IN DEPOSITS (LOW COST FUNDING)

HIGHER INTEREST PAID

DECREASE IN NET COMMISSIONS

INCREASE IN EXPENSES

P&L

CET1 CAPITAL
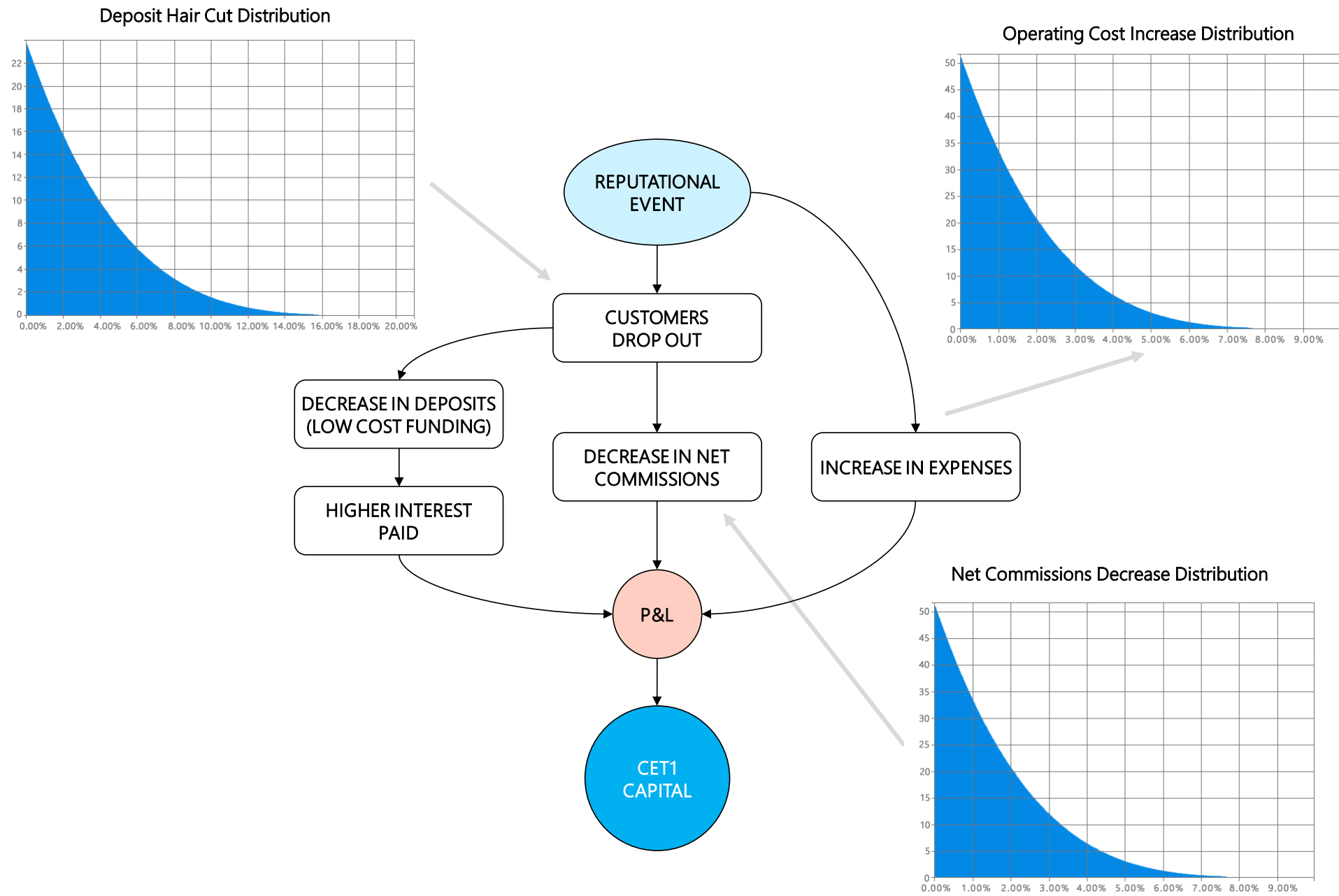
The Basic Idea Behind Stochastic Simulated Annealing

- **Reverse stress test problem** solving can be compared to a **scenario optimization issue**.
- In the **complex context** characterized by a **multiperiod** forecast model, relevant **non-linear conditions** and further **stochastic variables linked to idiosyncratic risk** (lower layer of the model), the choice of the most appropriate quantitative technique is pivotal (i.e. we have more than a variable to optimize).
- We propose as optimization system the **Simulated Annealing** (SA) **Driven by Multi-Start Strategy**, an iterative heuristic aimed at approximating a global optimization in a large search space.
- To adapt the SA to a stochastic simulation framework, including further stochastic variables in addition to those to be optimized, **we performed n trials for each step**, thus obtaining a more accurate average value for establishing whether a point should be added to the set of breaking point solutions or not.

PSEUDOCODE STOCHASTIC SIMULATED ANNEALING

loop over grid points
    initialize T (iterations/temperature)
    loop over the number of T
        pick a random solution x_c from the distribution and perform n trials for $f(x_c)$
        compute E_c as average of the n trials
        if $E_c < E_s$ **then** store x_c as new minimum and E_c as E_s
        else if $e^{-(E_s - E_c)}/T >=$ random(0, 1) **then** set x_c as x_s
     Reduce T
    end loop
end loop

Advantages

- SA enables us to **reach an optimal solution reducing the amount of sampling** necessary; allowing us to **move more quickly towards the breaking condition**, detecting the scenario that presents the highest probability of lying on the edge of the default area.
- SA allows us to better **guide the search by setting a calibration of the optimization process**; adjusting the system to the particular purpose of the analysis by properly setting the range of potential values of the variables to be optimized and the steps of the search process.
- SA is characterized by an adequate **balancing of the trade-offs between accuracy and computational efficiency**.

- In order to detect one Reverse Stress Test scenario from among all those associated to the several breaking points solutions determined by the optimization process, we need to apply a **criterion of selection**.

- At this aim we preliminary need to produce some **statistical metrics necessary** for any reasonable choice, which will help us in **synthesizing the multidimensional complexity of the information embedded into the break-even** points (percentiles, absolute mean deviation, etc.).

- We propose to adopt the **criterion of proximity** to select the reverse stress test scenario; **that is the closest scenario to the current market conditions** and in our opinion is the most sensible choice, since it may be considered somehow the most likely to occur, or at least the scenario that may trigger the breach before others.

- The criterion of proximity can be applied by determining the **risk factors combination that minimizes the distance from the origin of the break even point solutions** determined .

- We suggested as metric a particular case of **Euclidean distance and the Mahalanobis distance**.

- We performed a reverse stress test exercise based on an aggregated sample of **the four largest Italian banks**: Intesa Sanpaolo, Unicredit, Banco BPM and UBI Banca, representing in terms of total assets slightly more than **50% of the Italian banking industry**.

- To create the banks' sample, we added up all the banks' financial statement items so as to create a sort of aggregated balance sheet which we called ITB (Italian Bank), which can be considered as representing a typical Italian bank or a rough proxy of the Italian banking industry.

- The exercise time horizon is 2019-2021, considering 2018 financial statement data as the starting point.

- For safe of simplicity we adopted a **static balance sheet assumption**, anyway this assumption is not necessary and can be easily removed.

- **Systemic Risk factors** considered in the optimization process are:
  – **Credit Risk**, through PD/LGD and IFRS9 modeling, driven by **GDP** as stochastic variable.
  – **Market Risk**, driven **by EURO STOXX 50** index and its volatility as stochastic variables.
  – **Sovereign Risk**, driven by the **BTP-BUND Spread** as stochastic variable.
  – **Interest rate Risk**, driven by **Euribor** as stochastic variable.

- Further **Idiosyncratic Risk** factors considered in the modeling framework are:
  – **Operational Risk.**
  – **Reputational Risk.**

- The table below shows the **range of potential values** for each systemic risk driver adopted **in the reverse stress test optimization process**.

SYSTEMIC RISK FACTOR	VALUES AT: 31 Dec 2018	MIN	MAX
Italian GDP rate of change	0.9% (*)	-2.0%	0.0%
10-year BTP-Bund Spread	250 BPS	450 BPS (+200 BPS)	650 BPS (+400 BPS)
Euribor Swap Rate 6M (**)	-0.237%	0.32% (+0.557%)	0.62% (+0.857%)
SX5E Index rate of change (***)	3,001	1,800 (-40% rate of chg.)	2,701 (-10% rate of chg.)
SX5E Volatility (***)	12.61%	25.00% (+12.39%)	45.00% (+32.39%)

DISCLAIMER: The stress test exercise performed has been developed exclusively as an exemplification for illustrative purposes and does not represent to any extent a valuation on the capital adequacy of the banks considered.

- The **probability of breaching the TSCR** threshold is null in the first two years and extremely low in the third year.

- While, as to be expected, the cumulated **probability of breaching the higher OCR** threshold is extremely high in the third year, substantial in a two-year time period and negligible in a one-year time period.

- The results of the **probability of breach** stress test indicate that a reverse stress test scenario **can be determined only in the third year (2021) for the TSCR threshold** (since there are no breaches in the first two years), and **for all the three forecast years for the OCR threshold**.

Probability of Breach of CET1 Ratio: TSCR 6.5% Threshold

	2019	2020	2021
Marginal Probability	0.000%	0.000%	0.1012%
Cumulated Probability	0.000%	0.000%	0.1012%

Probability of Breach of CET1 Ratio: OCR 9.54% Threshold

	2019	2020	2021
Marginal Probability	0.069%	35.492%	61.561%
Cumulated Probability	0.069%	35.561%	97.053%

Reverse Stress Test Scenario – 2021 (113 Break-Even Scenarios): TSCR 6.5% Threshold
(Average Values: 2019-21)

	GDP	Δ% BTP/BUND SPREAD	EURIBOR SWAP RATE	YoY% SX5E VALUE	SX5E VOL (360D)
MEAN	-1.86%	3.70%	0.52%	-8.63%	36.86%
MEAN.DEV	0.05%	0.19%	0.07%	2.48%	4.87%
95% PERCENTILE	-1.74%	3.94%	0.61%	-4.50%	43.67%
5% PERCENTILE	-1.96%	3.26%	0.36%	-12.93%	26.99%

Risk Factors Impact – 2021 (113 Break-Even Scenarios): TSCR 6.5% Threshold
(Average Rating: 2019-21 & Cumulative Million Values: 2019-21)

	SOVEREIGN RATING	BANK RATING	PILLAR 1			PILLAR 2			CUMULATIVE NET TOTAL LOSS
			CREDIT RISK	MARKET RISK	OPERATIONAL RISK	SOVEREIGN RISK	INTEREST RATE RISK	REPUTATIONAL RISK	
MEAN	B+	B+	-58,026	-728	-2,580	-15,713	-4,333	-1,663	-39,506
MEAN.DEV	-	-	953	677	970	947	259	429	585
95% PERCENTILE	BB	BB-	-55,880	455	-962	-13,225	-3,760	-723	-38,481
5% PERCENTILE	B+	B+	-59,692	-2,199	-4,481	-16,993	-4,734	-2,460	-41,099

Reverse Stress Test Scenario – 2021 (259 Break-Even Scenarios): OCR 9.54% Threshold
(Average Million Values: 2019-21)

	GDP	Δ% BTP/BUND SPREAD	EURIBOR SWAP RATE	YoY% SX5E VALUE	SX5E VOL (360D)
MEAN	-0.50%	3.03%	0.47%	-8.12%	35.13%
MEAN.DEV	0.21%	0.48%	0.08%	2.53%	4.65%
95% PERCENTILE	-0.14%	3.83%	0.61%	-3.67%	43.34%
5% PERCENTILE	-0.87%	2.17%	0.33%	-13.13%	26.58%

Risk Factors Impact – 2021 (259 Break-Even Scenarios): OCR 9.54% Threshold
(Average Rating: 2019-21 & Cumulative Values in Million : 2019-21)

	SOVEREIGN RATING	BANK RATING	PILLAR 1			PILLAR 2			CUMULATIVE NET TOTAL LOSS
			CREDIT RISK	MARKET RISK	OPERATIONAL RISK	SOVEREIGN RISK	INTEREST RATE RISK	REPUTATIONAL RISK	
MEAN	BB	BB-	-39,516	-500	-2,267	-12,297	-4,109	-1,504	-18,574
MEAN.DEV	-	-	2,293	660	838	2,510	294	384	1,264
95% PERCENTILE	BB+	BB	-35,789	768	-760	-8,222	-3,581	-780	-16,483
5% PERCENTILE	B+	B+	-43,601	-1,736	-4,017	-16,668	-4,649	-2,316	-20,877

Data Elaboration: by Streeling.Simulator.

Reverse Stress Test Scenario – 2020 (333 Break-Even Scenarios): OCR 9.54% Threshold
(Average Values: 2019-20)

	GDP	Δ% BTP/BUND SPREAD	EURIBOR SWAP RATE	YoY% SX5E VALUE	SX5E VOL (360D)
MEAN	-1.42%	3.12%	0.48%	-41.66%	35.62%
MEAN.DEV	0.30%	0.49%	0.08%	2.39%	4.74%
95% PERCENTILE	-0.83%	3.89%	0.62%	-37.04%	43.74%
5% PERCENTILE	-1.90%	2.21%	0.34%	-46.40%	26.62%

Risk Factors Impact – 2020 (333 Break-Even Scenarios): OCR 9.54% Threshold
(Average Rating: 2019-20 & Cumulative Values in Million : 2019-20)

	SOVEREIGN RATING	BANK RATING	PILLAR 1			PILLAR 2			CUMULATIVE NET TOTAL LOSS
			CREDIT RISK	MARKET RISK	OPERATIONAL RISK	SOVEREIGN RISK	INTEREST RATE RISK	REPUTATIONAL RISK	
MEAN	BB	BB-	-32,384	-639	-1,590	-12,584	-2,328	-1,231	-18,928
MEAN.DEV	-	-	1,983	530	619	2,529	283	321	1,294
95% PERCENTILE	BB+	BB	-28,563	481	-434	-8,292	-1,779	-540	-16,829
5% PERCENTILE	B+	B+	-35,711	-1,819	-2,944	-16,582	-2,857	-1,871	-21,228

- In the 1-year breach reverse stress test, focused on 2019, the optimization process found only one breaking point solution; indicating a very low likelihood of breaching the threshold in a time period of just one year.

Reverse Stress Test Scenario – 2019 (1 Break-Even Scenario): OCR 9.54% Threshold

GDP	Δ% BTP/BUND SPREAD	EURIBOR SWAP RATE	YoY% SX5E VALUE	SX5E VOL (360D)
-1.90%	6.38%	1.21%	-32.58%	42.42%

Risk Factors Impact – 2019 (1 Break-Even Scenario): OCR 9.54% Threshold
(Values in Million)

SOVEREIGN RATING	BANK RATING	PILLAR 1			PILLAR 2			NET TOTAL LOSS
		CREDIT RISK	MARKET RISK	OPERATIONAL RISK	SOVEREIGN RISK	INTEREST RATE RISK	REPUTATIONAL RISK	
B+	B+	-17,571	-2,097	-406	-16,031	-1,988	-4,673	-17,122

- Limiting the application of the Criterion of Proximity to only the **two main risk factors GDP and BTP-Bund spread** – which **cover more than 80% of the impact on CET1 ratio** – we can plot all the combinations of values associated with the **113 breaking points found** through the optimization system for the 6.5% TSCR threshold reverse stress test.

- The **red dot indicates the combination of GDP and spread changes that minimizes the Euclidean distance** from the origin (starting point market conditions) among all the breaking points.
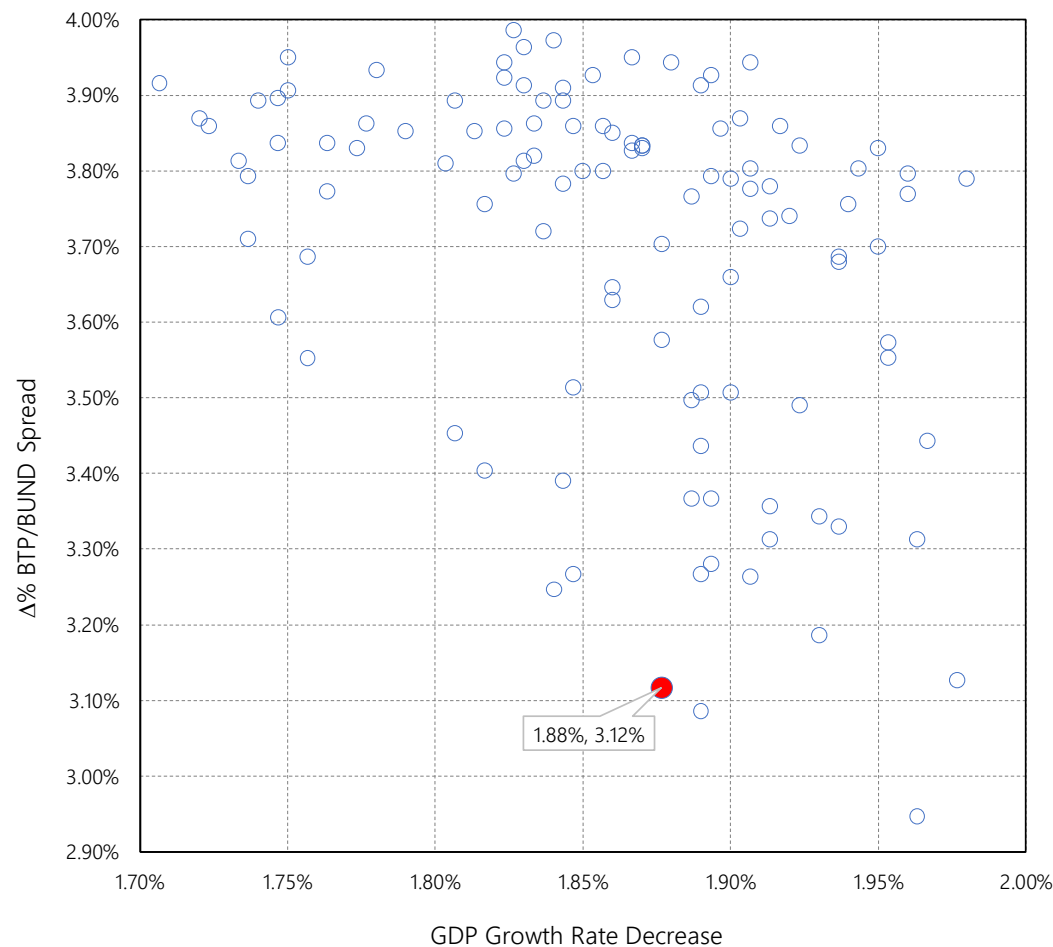
- The reverse stress test technique presented is a practical and manageable risk assessment approach, suitable for both **micro and macro prudential analysis**.

- The proposed framework is quite **flexible** and allows the user to easily introduce **additional risk factors**, and **more refined satellite models** and a greater **break-down of variables**, providing a practical and effective solution to a very challenging computational problem.

- The same methodology can also be applied to **calibrate early warning thresholds** for key risk indicators, by selecting the risk factor variable (KRI) to be optimized and by properly setting the relevant threshold to be triggered in the reverse analysis process

- We also presented a possible way to model some relevant **Pillar 2 risks** and their **interlinkage, feed-back and second round effects**, such as **sovereign, interest rate and reputational risks**.

- The methodological approach presented is well suited to be applied by **banks' risk managers** and **supervisors** in all enterprise-wide bank risk assessment processes that require a reverse stress test exercise: **RAF, ICAAP, ILAAP, Recovery Plan, SREP**.

- Reverse stress testing can be useful to understand the sources of risk and the triggering levels of some primary risk drivers; albeit **for effectively assessing a bank's overall risk (financial fragility degree) we need to do something different: to estimate its probability of breach**.

THANKS FOR YOUR ATTENTION