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Taking regulation seriously: fire sales under solvency and liquidity constraints

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Abstract

We build a framework for modelling fire sales where banks face both liquidity and solvency constraints and choose which assets to sell in order to minimise liquidation losses. Banks constrained by the leverage ratio prefer to first sell assets that are liquid and held in small amounts, while banks constrained by the risk-weighted capital ratio and the liquidity coverage ratio need to trade off assets' liquidity with their regulatory weights. We calibrate the model to the UK banking system, and find that banks' optimal liquidation strategies translate into moderate fire-sale losses even for extremely large solvency shocks. By contrast, severe funding shocks can generate significant losses. Thus models focusing exclusively on solvency risk may significantly underestimate the extent of contagion via fire sales. Moreover, when studying combined funding and solvency shocks, we find complementarities between the two shocks' effects that cannot be reproduced by focusing on either shock in isolation.

Key words: Banks, financial regulation, fire sales, stress testing, systemic risk.

JEL classification: G18, G21.

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Contents

During the early 'liquidity phase' of the financial crisis that began in 2007, many credit institutions, despite maintaining adequate capital levels, experienced significant difficulties because they had failed to manage their liquidity risk prudently. Some credit institutions became overly dependent on short term financing which rapidly dried up at the onset of the crisis. Such credit institutions then became vulnerable to liquidity demands because they were not holding a sufficient volume of liquid assets to meet demands to withdraw funds (outflows) during the stressed period. Credit institutions were then forced to liquidate assets in a fire-sale which created a self-reinforcing downward price spiral and lack of market confidence triggering a solvency crisis.

(European Commission,[1] 2015)

1 Introduction

The global financial crisis showed that financial institutions can be forced to liquidate assets at distressed prices, amplifying and transmitting shocks across the wider financial system. The above quote from the European Commission sets out how liquidity risk played a key role in giving rise to fire sales during the crisis. In response, after the crisis regulators introduced new liquidity regulations as well as tightened capital requirements for banks. Whilst these regulatory changes are likely to have strengthened banks' balance sheets in normal times, they may also put constraints on banks that affect their behaviour in times of stress. In this paper we seek to understand the role that the post-crisis regulatory framework might have in triggering fire sales. The framework allows us to address important questions such as: Which types of financial shocks and regulatory requirements combine to produce fire sales? How do banks optimally liquidate their portfolios when they are forced to do so?

Alongside a more stringent regulatory framework, regulators have introduced regular and comprehensive stress testing frameworks to assess the financial system's ability to weather severe market stresses. There is widespread agreement on the need to make stress testing more macro-prudential through inclusion of feedback effects and amplification mechanisms, such as fire sales. While progress has been made in this direction,[2] existing models are too stylised to provide a realistic assessment of banks' defensive actions under stress. We aim to bridge this gap by building a flexible analytical tool for regulatory stress tests to assess the risks from fire sales.

[1] `https://publications.europa.eu/en/publication-detail/-/publication/d70dbd16-9e0e-11e4-872e-01aa75ed71a1/language-en`.

[2] For example the ECB recently published STAMP€(Dees and Henry, 2017) which describes their analytical framework for macroprudential stress testing, and the Bank of England recently published a description of the models used to capture feedback and amplifications (see Bank of England (2017a) and Bardoscia et al. (2017)).

With this goal in mind, we develop a quantitative model of fire sales, building on Greenwood et al. (2015) and Cont and Schaanning (2017). In these models banks deleverage by selling assets in proportion to their holdings in order to restore their leverage ratios following a shock to their solvency. However in practice, that choice will likely, at least in part, be driven by the expected liquidation losses, as well as by the trigger which forced the bank to act. We extend these models along three dimensions to capture these features. Firstly, we subject banks to both funding and solvency shocks. Secondly, we expand the set of regulations banks face to include leverage, risk-weighted capital and liquidity regulations. Thirdly, we allow banks to optimally choose which assets to sell in order to minimise the losses they incur whilst deleveraging. We calibrate the model to the UK banking system and study the impact of both solvency and funding shocks. We find moderate fire-sale losses due to binding capital requirements, even for extremely large solvency shocks. By contrast, severe funding shocks can generate significant losses. This can be mitigated if banks use their liquid asset buffers through a stress.

The model begins with a shock to banks' balance sheets associated with asset losses and/or funding outflows. Banks are subject to risk-weighted capital and leverage ratio requirements, together with the liquidity coverage ratio (LCR) of the Basel III regulations.[3] The shock may force banks to sell assets for two reasons. First, the funding shock might force banks to raise cash to pay out withdrawing creditors. Second, the shock might take banks below their capital or leverage requirements or below a level of the LCR that a bank may seek to maintain if it is reluctant to use its liquid asset buffer; which would force banks to sell assets in order to deleverage their balance sheets and restore these ratios. Such distressed asset sales generate price impacts, which impose losses on other holders of the same assets.

We assume that any bank that needs to sell assets will do so optimally. In particular, it will seek to minimise the losses it incurs due to its liquidations, subject to these sales being sufficient to restore its regulatory ratios above their minimum requirements and pay out withdrawing creditors. As a result of the price impact of sales, banks are incentivised to sell liquid assets, as characterised in our model by high market depths, and assets which they do not hold in large amounts. However, banks must balance these concerns against the fact that the regulatory weights in the LCR and risk-weighted capital ratio generally make selling illiquid assets more effective in improving these ratios.

Our study yields a number of interesting findings. First, by deriving analytical solutions under a simplified version of the model we are able to establish simple and intuitive optimal liquidation strategies. In particular, we show that a bank subject only to a leverage ratio requirement would primarily care about the liquidity of different assets: it would first sell assets which have a more liquid secondary market and that it does not hold in large quantities. Specifically, it would sell assets sequentially

[3] We do not include the Net Stable Funding Ratio, as it had not been implemented in the UK at the end of 2016, which is the period we use to calibrate the model.

in descending order of their ratio of market depth to holdings size. In contrast, a bank faced with a risk-weighted capital ratio constraint or that wishes to maintain a high LCR through a stress must balance the liquidity of an asset with its weight in these two regulatory ratios. As a result, it would sell assets sequentially based on the market depth to holdings size ratio weighted by the relevant regulatory weights.

Second, using a calibration for the UK banking system, we evaluate the consequences of these liquidation strategies under given stress scenarios. In particular, we first analyse fire sales due to solvency shocks, based on variants of the Bank of England's 2017 stress test scenario. We find that losses are mainly driven by the risk-weighted capital ratio, which leads banks to sell larger quantities and less liquid assets relative to the leverage ratio. Nonetheless, aggregate fire-sale losses due to solvency shocks and binding capital requirements remain moderate even for extremely large shocks.

We then study fire sales due to funding shocks, based on variants of the outflow scenario envisaged in the calibration of the LCR. The model generates significant fire-sale losses following severe shocks. However, losses are reduced when banks consider their liquid asset buffers to be fully usable, relative to the case when they are unwilling to use them. This is because in the former case banks make use of their cash reserves and most liquid assets first, while in the latter case they liquidate assets that have a low impact on their liquid buffers and are thus less liquid.

Although funding and solvency shocks are fundamentally different, it is instructive to compare the results from the two stress scenarios. Losses due to funding shocks can be up to four times larger than the largest losses following a solvency shock. This is partially due to banks' optimal liquidation strategies. In particular, following a solvency shock, there are some assets - those that are illiquid and they hold in large amounts - that banks would never find optimal to sell, as doing so would not improve their solvency. By contrast, for extreme funding shocks banks need to raise cash to pay their creditors, and will in extremis sell any asset to do so, even if it causes large losses. As a result, the set of assets banks are willing to sell is greater for a funding shock than a solvency shock, leading to larger fire-sale losses. The results highlight the potential of liquidity risk to generate significant risk of fire sales, a channel overlooked by existing models that focus exclusively on solvency risk.

We then study combined funding and solvency shocks, of the type many distressed banks faced in the crisis. Fire-sale losses under combined funding and solvency shocks are larger than the losses generated by either of the two shocks in isolation, as banks' vulnerability and responses to these two type of shocks are heterogeneous. But losses under the combined shock are smaller than the sum of the losses generated by the two shocks considered in isolation, as banks' responses to funding and solvency shocks can be complementary. This underlines the importance of considering solvency and funding shocks together, as running the two stress scenarios independently may lead one to under- or over-estimate fire-sale losses.

Finally, we use the model to draw a number of interesting policy implications. The first relates to the usability of banks liquid asset buffers. The LCR ensures banks hold sufficient liquid assets to cover net liquidity outflows for a severe but plausible stress for a minimum of 30 days. The stock of liquid assets 'is intended to serve as a defence against the potential onset of liquidity stress' and banks can fall below 100% in a stress, avoiding fire sales.[4] Nevertheless, banks may in certain circumstances be reluctant to use some or all of their liquid asset buffers. Our model shows that usability of liquidity buffers in stress is key to reducing fire-sale losses. Secondly, we show that the UK leverage framework, whereby cash and central bank reserves are excluded from the calculation of banks' leverage ratios, could have the unintended consequence of increasing fire-sale losses in certain stress events. The UK framework may reduce banks' ability to boost their leverage ratios by using central bank reserves to deleverage, potentially forcing them into fire sales for smaller initial shocks.

1.1 Literature review

The literature on fire sales is vast, and we do not attempt to synthesize it here; we refer to Shleifer and Vishny (2011) for a broad survey of the literature. Instead, we review the papers that most closely relate to the key features of our model: what triggers fire sales, how banks decide which assets to sell, and the interaction between bank solvency and liquidity regulations.

At the heart of every model of fire sales lies the question of what forces agents to act. This can be "hard" constraints, such as breaching a solvency requirement (Cont and Schaanning, 2017) or facing creditor withdrawals (Calimani et al., 2017). Or it can be "softer" market constraints, such as protecting desired buffers above regulatory minima, or increased risk aversion that leads to precautionary actions such as liquidity hoarding (Acharya and Merrouche, 2013). Most of the existing quantitative models of fire sales assume that the only constraint banks face is a leverage constraint, and banks sell assets when in violation of the constraint (Cont and Schaanning, 2017), or to target their pre-shock leverage[5] (Greenwood et al., 2015). This comes at the cost of realism, but brings simplicity - they can ignore the riskiness of assets, as well as the liability side of the balance sheet. In these studies fire sales can only be triggered by asset shocks and the liquidity pressures that banks face are ignored.

In light of the evidence on the linkages between liquidity and solvency pressures (Hellwig (2009), Gorton and Metrick (2012), Pierret (2015), Pérignon et al. (2017)), these studies therefore discard a potentially fundamental channel of fire-sale risk. In this paper we aim to bridge this gap by developing a model where banks face shocks to both assets and liabilities, and where both liquidity and capital regulations are in

[4] Basel Committee on Banking Supervision (2013).

[5] Duarte and Eisenbach (2013) have generalised Greenwood et al. (2015) to allow banks to partially adjust back toward a leverage target with time variation both in the target and adjustment speed.

place. To the best of our knowledge we are the first to develop a quantitative model to analyse price-mediated contagion when banks face multiple constraints, appropriately calibrated to reflect the current regulatory framework.

The second key modelling assumption is how agents choose which assets to sell when fire selling assets. Here, a common assumption in the literature is that banks deleverage by selling assets proportionally to their holdings, or follow other simple heuristics (Greenwood et al. (2015), Cont and Schaanning (2017)). In practice, however, banks are likely to optimise over which assets they sell, given the constraints they face. Braouezec and Wagalath (2018) formally derive the optimal liquidation policy for a bank in a stylised model with two assets and show that when there are no market frictions and no price impact it is optimal to liquidate assets with the highest risk weights first. However, when there is a moderate price impact, the optimal liquidation strategy solves a complex trade-off involving several parameters, such as the weight of each asset and the price impact.[6] We take this approach further by formalising the optimisation problem when banks have multiple asset classes to choose from and face multiple regulatory constraints.

Closely related to our model, Kirti and Narasiman (2017) study how the likelihood of fire sales is affected by liquidity and capital constraints. In line with our results, they find that while a leverage requirement leads to the sale of relatively safe assets, a risk-based constraint can push them towards more risky assets. Moreover, they show that the interaction of liquidity and capital requirements can incentivise banks to increase in size and as a consequence increase the likelihood of fire sales. However, as in their model sales occur at fair value and do not involve a price impact, it is not suited to studying price-mediated contagion.

Our findings fit within a broader literature assessing the effects and interactions of the various regulatory requirements faced by banks after the crisis (Cecchetti and Kashyap (2018), Cetina (2015)). Studying the (interdependent) effects of capital and liquidity regulation on banks' stability is an important topic that has not yet received much attention. Our results on how solvency and liquidity regulations affect fire sale risks contribute to this literature and the ongoing debate on how to consolidate and build on post-crisis regulatory reforms.

The rest of the paper is organised as follows. In Section 2 we set up the general model and then derive banks' optimal strategies in a simplified setup. Section 3 describes the data and how the model has been calibrated. In Section 4 we discuss the main empirical results of the paper. Section 5 discusses some potential extensions of the model and Section 6 concludes. The Appendix contains proofs of the analytical results and further details on data sources and model calibration.

[6]When the price impact is large, it may be impossible for banks to restore their capital ratios by deleveraging.

2 Model

The model takes as its input banks' initial balance sheets and a given stress scenario, as described in Sections 2.1 and 2.2 respectively. The scenario defines the shock to banks' initial balance sheets, taking the form of either a funding shock or a solvency shock. If a shock is sufficiently large to cause a breach in some banks' capital or liquidity constraints or banks need to raise cash to meet outflows, these banks will adjust their balance sheet by selling assets and retiring liabilities if needed. The impact of deleveraging is described in Section 2.3. Banks' reactions and optimisation problems are formally set out in Section 2.4. Finally, banks' optimal liquidation strategies are discussed in Section 2.5.

2.1 Balance sheets and regulatory constraints

There are N banks in the economy. Each bank i's assets consist of marketable securities $M_i = (M_{i,1}, \ldots, M_{i,K_M})^\top$, cash and cash-like assets c_i and non-tradable assets o_i. Where convenient, we gather marketable securities and non-tradable assets into a single vector $A_i = (A_{i,1}, \ldots, A_{i,K_A})^\top$. Bank i's liabilities are denoted $L_i = (L_{i,1}, \ldots, L_{i,K_L})^\top$, and its equity capital e_i.

Banks are subject to capital, liquidity and leverage constraints. We base these constraints on the Basel III framework.[7] Bank i's risk-weighted assets are given by:

$$\sum_k M_{i,k}\rho_k^M + o_i\rho^O = \sum_k A_{i,k}\rho_k = \rho^\top A_i,$$

where ρ denotes the risk weights associated to different assets. Note that cash holdings carry a risk-weight of 0. The risk-weighted capital ratio of bank i is then given by:

$$CAP_i := \frac{e_i}{\rho^\top A_i}. \tag{1}$$

Banks are required to ensure that $CAP_i \geq \beta_i^{CAP}$, where β_i^{CAP} can represent the minimum capital requirement for bank i (≈ 0.08 under Basel III) but it can also include some regulatory or voluntary buffers of capital.[8]

[7] See `https://www.bis.org/bcbs/basel3.htm`.

[8] It remains an open question how usable buffers actually are in stress and what level of capital banks would like to defend. Banks are currently building large buffers and are disincentivised to use them because of restrictions on bonus and dividend payments and potential market stigma once they dip into their regulatory buffers. In Section 4.1, when illustrating the model for the UK banking system, we follow the Bank of England stress testing framework and assume that banks defend their minimum capital requirements plus their systemic buffers for those banks designated as globally

Banks are also subject to a leverage requirement, whereby their ratio of equity to assets cannot fall below a given value: $LEV_i \geq \beta_i^{LEV}$, where analogously β_i^{LEV} can represent the minimum leverage ratio requirement for bank i (0.03 under Basel III) but it can also include some regulatory or voluntary buffers. The leverage ratio is defined as:

$$LEV_i := \frac{e_i}{\mathbf{1}^\top A_i + c_i}. \tag{2}$$

Note that the equity e_i refers to Tier 1 capital in the case of the leverage ratio, while it refers to Common Equity Tier 1 in the case of the risk-weighted capital constraint.

Finally, banks are subject to liquidity regulation. We base our liquidity requirement on the liquidity coverage ratio (LCR) of Basel III, where in normal times banks are required to ensure that they hold enough high quality liquid assets (HQLA) to meet net outflows under a 30-day stress scenario. Assets are grouped into liquidity buckets, with weights λ_k, which reflect the degree of liquidity of the asset. The HQLA of bank i are given by its cash holdings plus $\sum_k M_{i,k}\lambda_k$. Outflows are computed by weighting the liabilities by outflow weights, ω_k^{out}, and the inflows are computed in a similar manner by assigning inflow weights, ω_k^{in}, to the assets. The liquidity coverage ratio of bank i is then defined as the ratio of its HQLA to its net outflows:

$$LCR_i := \frac{\lambda^\top M_i + c_i}{\omega_{out}^\top L_i - \omega_{in}^\top M_i}.$$

We assume that banks aim to maintain their LCR above a certain level β_i^{LCR}. Banks are required to keep their LCRs above 100% in normal times, but liquid buffers are intended to be usable in times of stress to ensure that they have sufficient liquid assets to meet the net liabilities that might come due over a 30-day horizon. Banks may see some or all of the liquid asset buffer as not being usable in a stress. For this reason, when bringing the model to data, we will show results both for $\beta_i^{LCR} = 100\%$ and also for smaller values, including the case where the liquid asset buffer is fully usable as intended by the regulator ($\beta_i^{LCR} = 0$).

2.2 Stress scenario

The shocks in our model can hit both the asset and the liability side of the balance sheet. Asset shocks, $\epsilon_A \in [0,1]^{K_A}$, are defined as percentage losses on the different asset classes. Analogously, a funding shock is defined as a percentage loss in funding (e.g. redemption), $\epsilon_L \in [0,1]^{K_L}$, in the various liability classes. Post-shock, the state of the economy is described by banks' balance sheets and their regulatory ratios relative to their requirements.[9]. Asset shocks reduce the value of asset holdings to

systemically important.

[9]We take the post-shock economy to be time 0 in our model, and denote post-shock quantities with superscript 0

$A^0_{i,k} = A_{i,k}(1 - \epsilon^A_k)$ causing a corresponding reduction in equity equal to

$$e^0_i = e_i - \epsilon_A^\top A_i.$$

As redemptions and other funding shocks such as the loss of wholesale funding must be met in cash, they cause a reduction in the liability in question, $L^0_{i,k} = L_{i,k}(1 - \epsilon^L_k)$, and a decrease in the cash holdings equal to

$$c^0_i = c_i - \epsilon_L^\top L_i.$$

The banks' regulatory ratios will update to reflect these new values for assets, liabilities and equity.

2.3 The impact of deleveraging

We assume that following the initial shock there will be no government bail-out, no extraordinary central bank liquidity provision, and that banks cannot take any action except using available cash reserves and selling assets. Should the shock therefore mean that banks face a shortfall in one or more of their regulatory ratios, or should they have faced large outflows on their liabilities, they will be forced to liquidate some of their tradable assets in order to meet their outflows and/or deleverage their balance sheet.[10]

Before formally introducing the optimisation problem that each bank i solves to determine which quantity $S_{i,k}$ of which asset k to sell, we need to define the impact of asset sales on market prices and banks' balance sheets. In what follows, we assume banks undertake asset sales simultaneously. Banks can in principle undertake several rounds of deleveraging. Here we describe the evolution of the system for a general time step t, such that iterating the deleveraging cascade for further rounds is a straightforward extension.

Asset prices. If the volume of assets sold is sizeable relative to their market depths, the sale of those assets can have an adverse impact on their price. We follow a large stream of literature and adopt a linear functional form to model the price impact. As shown in Huberman and Stanzl (2004), a linear price impact is the only functional form that is consistent with the no-dynamic arbitrage theory and excludes the existence of profitable round-trip trades. This is important insofar that the banks in our model will try to minimise losses incurred from fire sales. For a deeper discussion on

[10] In principle, banks could resort to other actions to respond to solvency or liquidity stress. For example, they could cut funding to other institutions or try to obtain new funding themselves. They could raise new equity, but this can be challenging in a crisis, or reduce lending, though this may operate over a longer time period than that which we're interested in. We are thus essentially assuming that we are in a world where banks have exhausted other more preferable options, and are left with liquidating assets as their best available option.

price impact modelling, we refer to Bouchaud (2010), Donier et al. (2015), or Cont and Schaanning (2017).

Under a linear price impact function, when banks sell quantities $S_{i,k}$ (in monetary units) of asset k its price moves from p_k^t to

$$p_k^{t+1} = p_k^t \left(1 - \delta_k^{-1} \sum_{i=1}^{N} S_{i,k}^t\right) = p_k^t \left(1 - \delta_k^{-1} q_k^t\right), \tag{3}$$

where δ_k is the market depth of the asset and the total quantity of asset sales in asset class k is given by $q_k^t := \sum_{i=1}^{N} S_{i,k}^t$. The market depth is a measure of the asset's market liquidity capturing the ability to transact in size without moving its price. We refer to Section 3.2 and the Appendix 7.2.2 for details on the empirical estimation of the market depths for our model and a brief discussion on alternative measures of market liquidity.

Asset holdings. The asset sales, and their associated price impacts, change the value of the banks' holdings. Their new value is given by

$$M_{i,k}^{t+1} = \underbrace{(M_{i,k}^t - S_{i,k}^t)}_{\text{Remaining holdings}} \times \underbrace{\left(1 - \delta_k^{-1} \sum_{i=1}^{N} S_{i,k}^t\right)}_{\text{Impact on remaining holdings}}.$$

We assume that $S_{i,k}^t \leq M_{i,k}^t$ such that the institutions cannot short securities for the purpose of deleveraging.

Fire-sale losses and equity. The loss for bank i on its remaining holdings is given by

$$\sum_{k=1}^{K} (M_{i,k}^t - S_{i,k}^t) \frac{q_k^t}{\delta_k}.$$

As it is unlikely that the bank will be able to monetise all of its assets at their book value during a stress scenario, it will not only suffer mark-to-market losses on its remaining holdings, but also suffer an "implementation shortfall" on the assets it sells. When banks simultaneously sell quantities $S_{i,k}$ of assets (for $k = 1, ..., K$), the realised loss on the liquidated assets is given by

$$\alpha \sum_{k=1}^{K} S_{i,k}^t \sum_{j=1}^{N} \delta_k^{-1} S_{j,k}^t,$$

for some $\alpha \in [0, 1]$. For instance, $\alpha = 0.5$ corresponds to assuming that, within a period, the asset sales take place uniformly over time and the price also declines uniformly over time. The most conservative assumption is to set $\alpha = 1$, which assumes all assets are sold at the final price. Henceforth, for simplicity, we set $\alpha = 1$, in which case the two types of losses add up to a fire-sale loss given by the linear function

$$FLoss_i^{t+1} = \sum_{k=1}^{K} \delta_k^{-1} \sum_{j=1}^{N} M_{i,k}^t S_{j,k}^t,$$

or in vector notation

$$Floss_i^{t+1} = (M_i^t)^\top D^{-1} q, \tag{4}$$

where $D = \text{diag}(\delta_1, \ldots, \delta_{K_M})$ and $q = (q_1, \ldots, q_N)^\top$. The fire-sale loss reduces the bank's equity, over and above any initial shock, by the corresponding amount:

$$e_i^{t+1} = e_i^t - FLoss_i^{t+1}. \tag{5}$$

Liabilities. Finally, liabilities are updated as follows

$$L_{i,k}^{t+1} = L_{i,k}^t - R_{i,k}^t. \tag{6}$$

where $R_{i,k}^t$ denotes the amount of asset k that bank i retires.

Higher rounds of the cascade. After losses from deleveraging are accounted for, banks may still be in breach of their constraints. This is because they incur unexpected losses due to other banks' sales. As a result, there may be further rounds of fire sales. During the fire sales cascade the system suffers at each round a fire-sale loss, which adds up to a total:

$$FLoss := \sum_{t=1}^{T} \sum_{i=1}^{N} FLoss_i^t,$$

where $T \leq +\infty$ is the stopping time of the deleveraging cascade. The cascade continues until all banks reach one of the following three states: either they have liquidated enough assets to restore their constraints, they have run out of liquid assets to sell and they are deemed illiquid, or they have suffered losses large enough such that they are deemed insolvent.

2.4 Bank optimisation problem

We now describe how banks choose which assets, and which quantities thereof, to sell in response to a shock. To ease notation, we drop all time subscripts in what follows.

The distressed liquidation of assets will cause losses to the institution selling the assets, as well as mark-to-market losses to all other institutions that hold the same assets. When deciding which assets to sell in order to return to their regulatory requirements and pay out running creditors, we assume banks take into account the price impacts their sales will have and seek to minimse the losses they incur due to their sales. This setup differs from the models of both Greenwood et al. (2015) and Cont and Schaanning (2017), where banks do not take into account the price impact of sales when deciding how much of each asset to sell.

We assume banks do not anticipate the price impact that arises from the sales of other banks. This assumption will be realistic if banks have relatively little information on the asset holdings of their peers. In principle, banks may have *some* information on their peers' balance sheets and behaviour, and could condition their actions on this information. Modeling this would require a game theoretic approach, which is beyond the scope of this paper.[11]

Formally, we assume that a bank's objective during forced liquidations is to minimise the fire-sale loss caused by its own deleveraging. Consequently, for sales $S_i \leq M_i$ and retirements $R_i \leq L_i$, they solve:

$$\min_{S_i, R_i} M_i^\top D^{-1} S_i, \tag{7}$$

subject to the constraints

$$CAP_i(S_i) \geq \beta_i^{CAP} \tag{8}$$

$$LEV_i(S_i, R_i) \geq \beta_i^{LEV} \tag{9}$$

$$LCR_i(S_i, R_i) \geq \beta_i^{LCR} \tag{10}$$

$$CASH_i(S_i, R_i) \geq 0. \tag{11}$$

Given bank i's sales S_i, the risk-weighted capital ratio is given by:

$$CAP_i(S_i) := \frac{e_i - M_i^\top D^{-1} S_i}{\rho_M^\top \left[(M_i - S_i) \circ (\mathbf{1} - D^{-1} S_i)\right] + \rho^O o_i}, \tag{12}$$

where $\circ$ denotes element-wise multiplication of vectors. The constraint describes how the sale of assets S_i leads to a decrease in risk-weighted assets to $\rho_M^\top \left[(M_i - S_i) \circ (\mathbf{1} - D^{-1} S_i)\right]$, while at the same time causing losses equal to $M_i^\top D^{-1} S_i$.

The leverage ratio is given by:

$$LEV_i(S_i, R_i) := \frac{e_i - M_i^\top D^{-1} S_i}{(M_i - S_i)^\top (\mathbf{1} - D^{-1} S_i) + c_i + S_i^\top (\mathbf{1} - D^{-1} S_i) - \mathbf{1}^\top R_i + o_i}. \tag{13}$$

In the risk-weighted capital constraint (12), the cash holdings and the liability retirements did not appear, as cash has a risk weight of zero. In the leverage ratio

[11]We refer to Braouezec and Wagalath (2019) for a fire sales model with complete information that gives rise to a game with strategic complementarities.

they do however appear, as converting assets into cash without retiring any liabilities does not shrink a bank's balance sheet. As such, in order to improve its leverage ratio a bank that sells assets will then have to use the proceeds to retire liabilities and thus shrink its balance sheet.

The liquidity coverage ratio is given by:

$$LCR_i(S_i, R_i) := \frac{\lambda^\top [(M_i - S_i) \circ (\mathbf{1} - D^{-1}S_i)] + c_i + S_i^\top(\mathbf{1} - D^{-1}S_i) - \mathbf{1}^\top R_i}{\omega_{out}^\top (L_i - R_i) - \omega_{in}^\top [(M_i - S_i) \circ (\mathbf{1} - D^{-1}S_i)]}. \quad (14)$$

For the LCR, the changes should also be fairly intuitive: (i) Cash and cash-like holdings increase by the net difference between assets monetised and liabilities retired: $c_i + S_i^\top(\mathbf{1} - D^{-1}S_i) - \mathbf{1}^\top R_i$; (ii) The HQLA further change depending on whether the bank sold more or less liquid assets: $\lambda^\top [(M_i - S_i) \circ (\mathbf{1} - D^{-1}S_i)]$; (iii) the outflows and inflows also change to reflect the new balance sheet composition: $\omega_{out}^\top (L_i - R_i)$ and $\omega_{in}^\top [(M_i - S_i) \circ (\mathbf{1} - D^{-1}S_i)]$ respectively.

Finally, we impose a cash-consistency constraint

$$CASH_i(S_i, R_i) = c_i + (\mathbf{1} - D^{-1}S_i)^\top S_i - \mathbf{1}^\top R_i, \quad (15)$$

which ensures that the liabilities a bank retires cannot exceed its initial cash holdings plus sales proceeds.

2.5 Bank strategies

In this section we analyse how banks optimally liquidate assets, depending on which constraints they face. We first establish which assets banks would never choose to liquidate when deleveraging their balance sheet, because doing so would in fact harm their balance sheet position. We then set out banks' optimal deleveraging strategies according to the constraints they face.

The general problem (7) subject to the constraints (8) - (11) cannot be solved analytically, and so in our empirical application we will rely on numerical solvers to compute the banks' strategies. In this section, in order to analytically derive banks' optimal strategies, we (a) consider each constraint in isolation; (b) continue to assume a full price impact on the assets sold ($\alpha = 1$), and (c) linearise the constraints. We term this linear setup the 'approximate problem'. This will most closely approximate the full setup when sales are small and when only one consraint is binding for a bank. In Appendix 7.3, we show that the analytical solutions we derive for the approximate problem very closely match the numerical solutions we obtain for the general non-linear problem.

2.5.1 When does liquidating assets not help?

First we consider the set of assets banks will consider liquidating, depending on which constraints they face, and which assets they will never sell as doing so would be counter-productive.

The liquidation of assets has simultaneous positive and negative effects on the regulatory ratios. For instance in (13), it is easy to see that a liquidation improves the leverage constraint by virtue of decreasing the denominator.[12] However, the deleveraging simultaneously harms the leverage ratio by decreasing the numerator through liquidation losses. Whether selling the asset will alleviate the bank's leverage constraint will depend on the relative sizes of these two effects, as well as the bank's leverage ratio requirement. In particular, it is straightforward to show that a sale will alleviate the bank's leverage constraint if the losses due to the sale are less than a fraction β_i^{LEV} of the reduction in assets due to the sale, where β_i^{LEV} is the bank's leverage ratio requirement. If for all possible liquidation sizes of an asset k the net effect of its sale leads to a deterioration of the balance sheet, the bank will never choose to liquidate the asset in question. The following proposition formalises this logic and extends it to the other constraints. Proofs can be found in the Appendix.

Proposition 1. *A bank i facing a binding leverage constraint will never liquidate asset k for which*

$$\frac{M_{i,k}}{\delta_k} \geq \frac{\beta_i^{LEV}}{1-\beta_i^{LEV}}.$$

A bank i facing a binding risk-weighted capital constraint will never liquidate asset k for which

$$\frac{M_{i,k}}{\delta_k} \geq \frac{\rho_k \beta_i^{CAP}}{1-\rho_k \beta_i^{CAP}}.$$

A bank i facing a binding LCR constraint will never liquidate asset k for which

$$\frac{M_{i,k}}{\delta_k} \geq \frac{1-(\lambda_k + \omega_k^{in}\beta_i^{LCR})}{\lambda_k + \omega_k^{in}\beta_i^{LCR}}.$$

A bank facing a binding cash constraint will be willing to liquidate any asset.

Figure 1 shows schematically the sets of assets whose sale would ease each of the constraints. The assets that banks will be willing to sell to meet any of their regulatory requirements form a subset of the assets they can sell to raise cash. The sets of assets whose sale would help improve the compliance with a regulatory constraint are in large part overlapping - selling any of a large number of assets will boost all three regulatory requirements. However, there are some differences stemming from the fact that the regulatory requirements assign different weights to different assets.

[12] When the leverage constraint is binding, received cash will be used to retire liabilities, implying that $c_i + S_i^\top(\mathbf{1} - D^{-1}S_i) - \mathbf{1}^\top R_i = 0$.

Selling assets with zero risk weights will not help a bank meet its risk-weighted capital requirement, whilst selling an asset where the sum of inflows and liquidity weights is greater than or equal to 1 will not help a bank meet its LCR. As such, the set of assets whose sale would help banks meet their leverage ratio is generally greater in size than the set of assets that help a bank meet its risk-weighted capital ratio or LCR constraint. The set of assets whose sale would not boost a bank's leverage ratio is small, but by definition will result in large losses if sold. Depending on their regulatory weights, selling these assets may or may not boost the risk-weighted capital or LCR constraints. As shown in Section 4, this will have important implications in terms of fire-sale losses for solvency and funding shocks.

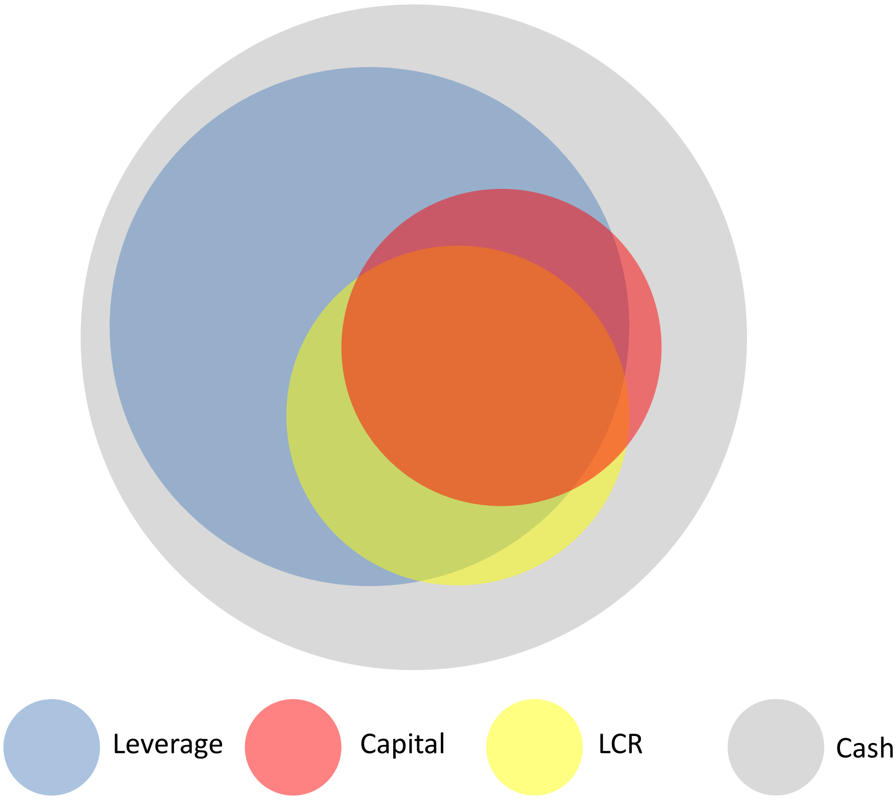


Figure 1: Venn diagram of assets whose sale will help a bank meet its constraints

2.5.2 How do banks optimally liquidate assets?

We now characterise banks' optimal liquidation strategies for the approximate problem. Intuitively, since banks aim to minimise liquidation losses, they are incentivised to sell liquid assets, and assets which they do not hold in large amounts. However, banks must balance these concerns against the regulatory weights imposed on assets in the LCR and risk-weighted capital ratio in order to sell assets that are most effective in improving these ratios. The following three propositions characterise the banks' optimal liquidation strategies when they are constrained by each of the three regulatory ratios in turn. Detailed proofs are relegated to the Appendix.

Leverage constraint. When banks face only a leverage constraint, the solution to the approximate problem is characterised by the following proposition.

Proposition 2. *The optimal strategy for a bank is to liquidate its assets sequentially, in the order given by the ratios*

$$\frac{\delta_{k_1}}{M_{i,k_1}} \geq \cdots \geq \frac{\delta_{k_{K_M}}}{M_{i,k_{K_M}}},$$

where the index k runs over the marketable assets $k = 1..K_M$. The bank will work through its assets in this order, selling all of each asset sequentially, until it has satisfied its constraint.

In the leverage ratio case, banks thus only consider the ratio of the asset's market depth to their holding size and prefer to sell assets that are liquid and which they hold in small quantities over assets that are illiquid and which they hold in larger quantities. The result will be a corner solution, where banks sell all their asset holdings where the ratio of market depth to holding size is above some critical value, and sell none of the assets where this ratio is below the critical value.

Risk-weighted capital constraint. When banks only face a risk-weighted capital constraint, they need to take into account the assets' risk weights as well as their liquidity. The solution to the banks' approximate problem is given by the following proposition.

Proposition 3. *The optimal strategy for a bank is to liquidate its assets sequentially, in the order given by the quantities*

$$\rho_{k_1}\left(1 + \frac{\delta_{k_1}}{M_{i,k_1}}\right) \geq \cdots \geq \rho_{k_{K_M}}\left(1 + \frac{\delta_{k_{K_M}}}{M_{i,k_{K_M}}}\right).$$

When the ratio $\frac{\delta_{k_j}}{M_{i,k_j}}$ is large, the strategy is approximated well by the ratios $\frac{\rho_{k_j}\delta_{k_j}}{M_{i,k_j}}$, which is similar to the strategy under the leverage constraint, except that the ratio is now weighted by the risk weight. This is intuitive, as for two assets for which the ratios $\frac{\delta_{k_j}}{M_{i,k_j}}$ are the same, a bank will prefer to sell the one with the higher risk weight.

LCR constraint. Similarly, when banks only face the LCR constraint, they need to account for the impact of the assets' sale on the ratio via their impact on HQLA and expected inflows. The solution to the banks' approximate problem is given by the following proposition.

Proposition 4. *The optimal strategy for a bank is to liquidate its assets sequentially, in the order given by the quantities*

$$\frac{\delta_{k_1}}{M_{i,k_1}}\left(1-\omega^{in}_{k_1}\beta^{LCR}_i-\lambda_{k_1}\right)-(\omega^{in}_{k_1}\beta^{LCR}_i+\lambda_{k_1})\geq\cdots$$
$$\geq\frac{\delta_{k_{K_M}}}{M_{i,k_{K_M}}}\left(1-\omega^{in}_{k_{K_M}}\beta^{LCR}_i-\lambda_{k_{K_M}}\right)-(\omega^{in}_{k_{K_M}}\beta^{LCR}_i+\lambda_{k_{K_M}})$$

To alleviate their LCR constraint, banks will thus trade off the liquidity of the assets that they sell, $\frac{\delta_{k_j}}{M_{i,k_j}}$, against the loss that the sale of said assets causes to the stock of HQLA and its inflows, $(1-\omega^{in}_{k_j}\beta^{LCR}_i-\lambda_{k_j})$. A bank will thus prefer to sell assets that incur a low price impact and that do not cause a large drop in its stock of HQLA or its expected inflows.

3 Data

To empirically illustrate how the model can be used to analyse risks of spillovers via fire sales of commonly-held securities, we have applied the model to the seven UK banks that are subject to the Bank of England's annual stress test exercise.[13] In this section we briefly describe the data and provide the key summary statistics. More technical details on calibration and data sources are provided in the Appendix.

3.1 Balance sheets and regulatory ratios

Assets and liabilities. Using granular supervisory data sources from end 2016 – Financial Reporting Framework (FINREP), Common Reporting Framework (COREP) and instrument-level securities holdings from the Bank of England's confidential stress test reports – we build the balance sheets of the seven banks in our model. We divide banks' assets into tradable ($M_{i,k}$) and non-tradable assets (o_i). Tradable assets are disaggregated at three levels:[14]

[13]These are in alphabetical order Barclays, HSBC, Lloyds Banking Group (LBG), Nationwide, Royal Bank of Scotland (RBS), Standard Chartered Bank (SCB), Santander UK (SanUK).

[14]We consider tradable assets to be assets that, for accounting purposes, are recorded as *held for trading*, *available for sale* or *held at fair value.*

1. *Asset type:* Cash, debt (sovereigns), debt (financial corporates), debt (non-financial corporates), equity.

2. *Geographical region:* Belgium, Canada, China, Finland, France, Germany, Hong Kong, Italy, Japan, Korea, Netherlands, Other, Singapore, Spain, Sweden, UK and US. These are the 17 regions in which UK banks have the largest securities holdings.

3. *Liquidity:* Regulatory data allows us to categorise different securities according to their LCR classification.

We classify as non-tradable assets all other assets comprising loans, residual assets (e.g. intangible goods) and derivatives. This assumption is made for simplicity, as quantifying the costs of deleveraging these assets would be challenging and it is outside the scope of this paper. Overall, this leads to 227 different asset classes on banks' balance sheets.

On the liability side, we use FINREP balance sheet data and LCR outflow data from COREP which provides a classification of liabilities based on their outflow weights.

Stylised facts. The 10 largest regions for sovereign bond holdings across all UK banks are UK (24.3%), US (23.2%), Japan (8%), Germany (5.3%), France (4%), Singapore (2.5%), South Korea (2.2%), Italy (1.6%), Hong Kong and China.[15] Debt issued by financial corporates is held as follows: 55.7% in US denomination, 16.9% in GBP, 15.6% in EUR and 11.9% in other denominations. Debt issued by non-financial corporations is held as follows: 43.1 % in USD, 14.6% in EUR, 13.6% in GBP and 28.1 % in other currencies. Finally, equity holdings are split as: 42.8% in USD, 30.8% in EUR, 24.9 % in GBP and 1.5 % in other currencies. In terms of asset classes, the largest holdings are in sovereign bonds (61.2%), followed by debt issued by non-financial corporates (30.3%), in turn followed by financial corporates' debt (4.9%) and equities (3.6%).

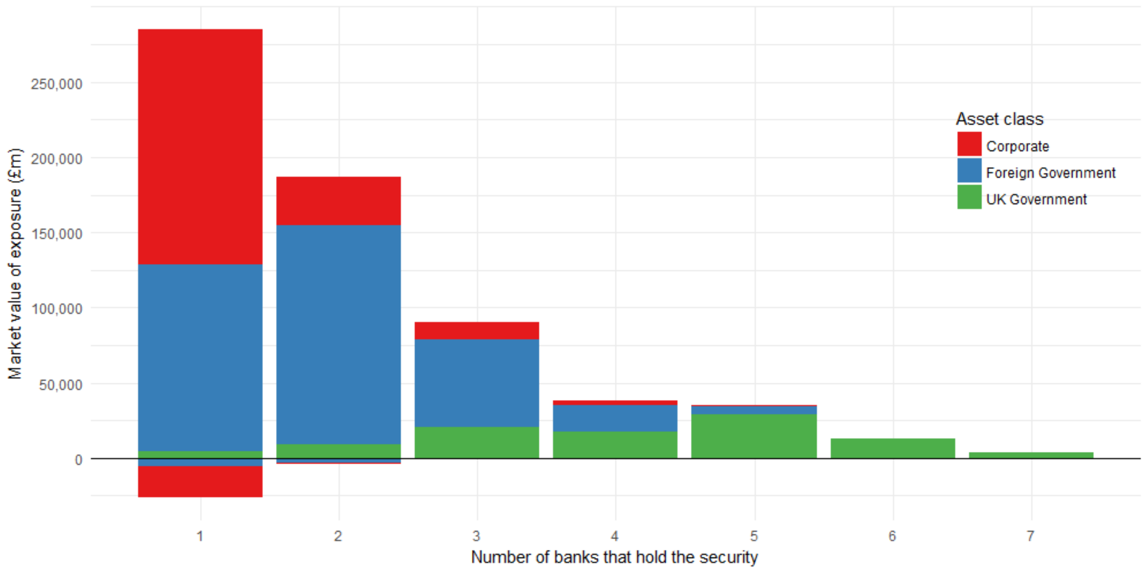
The instrument-level data allows us to get a clearer picture of the degree of common asset holdings in the different tradable assets' categories.[16] Unsurprisingly, Figure 2 shows that UK government bonds is the asset class most commonly held by the seven stress test banks, followed by foreign government bonds and corporates. For example, the majority of UK government bonds are held by four or more stress test banks; while overlaps in individual corporate bonds are mostly limited to three firms or fewer. UK government bonds are a highly liquid asset class, and UK banks only held around 5% of the total amount outstanding (£ 1.8 tn) at the end of 2016.

[15] For confidentiality reasons the holdings of these regions cannot be reported.

[16] Note that the instrument-level dataset on tradable securities, collected for the 2017 stress test, has an optional materiality threshold whereby only holdings greater than £50 million need to be reported. The part of the bars below zero represents short positions.

This observation, together with the fact that the banks hold large amounts in cash and cash equivalents instruments due to liquidity regulation, suggests *a priori* that the risk of amplification through sales of commonly held assets should not be elevated within the UK banking system.[17] Nonetheless, the data provides an interesting laboratory to present the key predictions from the model and analyse how price-mediated contagion could play out when the UK banking system faces solvency and liquidity stress.

Figure 2: Market value of overlapping portfolio by asset class (ISIN level)



Regulatory ratios. Table 1 reports total assets, risk-weighted assets for market and credit risk, total risk-weighted assets, banks' CET 1 capital, and banks' regulatory ratios as of 31st December 2016. The capital and leverage ratios are taken from the Bank of England's stress test results (Bank of England, 2017a). All the seven stress test banks have LCRs above 100% as well as capital and leverage ratios above the regulatory requirements.[18]

[17] The Bank of England has reached similar conclusions in the 2017 stress test, adapting a methodology by Cont and Schaaning (2017). See Bank of England (2017a).

[18] Note that under the Pillar 3 disclosure framework banks must now disclose detailed quantitative information from their LCR regulatory return quarterly. However the first application of this disclosure requirement commenced at a bank's financial year-end 2017, hence we do not provide a breakdown of banks' LCRs in this paper. We refer to `https://www.bis.org/bcbs/publ/d400.pdf` for more information.

Table 1: Banks assets (£ bn)

	Barclays	HSBC	LBG	Nationwide	RBS	SanUK	SCB
Total Assets	1207	1870	677	225	808	303	535
Total RWA	366	857	216	34	228	44	269
RWA Market Risk	25	41.5	3.1	0	17	1.7	22
RWA Credit Risk	242	656	168	29.7	185.9	38	214
Capital (CET1)	45.2	116.6	29.3	8.6	30.6	10.2	36.6
Capital ratio (%)	12.4	13.6	13.6	24.4	13.4	11.6	13.6
Leverage ratio (%)	5	5.7	5.2	4.3	5.6	4.1	6
LCR (%)	131	136	n.d.	124	123	139	n.d.

Source: Bank of England (2017a) and annual reports. LBG and Standard Chartered have not publicly disclosed their LCRs for Q4 2016.

3.2 Market depths

The tradable assets' market depths, δ_k, determine the price impact associated with their liquidation, and as such they are fundamental parameters of the model. There is a vast literature on the measurement of (relative) liquidity of assets.[19] However, for the purpose of our model, it is necessary to determine absolute levels of liquidity that specify – as a function of the liquidation volume – by how much the prices of assets would decrease. This is a notoriously difficult question and so far no universally accepted methodology exists to address this task. We follow Cont and Schaanning (2017) and define asset k's market depth as the ratio between the average trading volume (ADV_k) and the daily volatility σ_k:

$$\delta_k(\tau) = c\frac{ADV_k}{\sigma_k}\sqrt{\tau}, \tag{16}$$

where c is a scaling parameter equal to 0.3 which is calibrated using transaction-level data[20] and τ is the liquidation horizon. As the average daily volume scales with τ, while the volatility scales with $\sqrt{\tau}$, the market depth as a whole scales with $\sqrt{\tau}$. This reflects the intuition that the price impact becomes smaller when one liquidates a position over a longer time horizon. The results presented in the next section have been obtained assuming a 5 day-liquidation horizon, in line with the week during which Bear Stearns ran out of funding during the global financial crisis (Duffie, 2010).

[19]For example Bao et al. (2011), Chen et al. (2007), Mahanti et al. (2008) and references therein.

[20]A theoretical motivation for this market depth measure can be found among the "market microstructure invariance" principles developed by Kyle and Obizhaeva (2016) and Obizhaeva (2012) for the empirical estimation of c using portfolio transition trades data.

We have estimated market depths from 2017 which we consider to be a period of normal market conditions. To simulate more stressed market conditions, we have also calibrated the market depth to data from 2008.[21] Figure 3 shows the distribution of the two market depth calibrations for 2017 and 2008 respectively; market depth estimates are clearly smaller in 2008 reflecting the stressed conditions compared to 2017. The median market depth across all tradable assets in 2017 implies a drop in prices of 0.6% (under a linear price impact) for a forced liquidation of 1 bn pound over 5 days, while the median market depth across all tradable assets in 2008 implies a 4% decrease. The main results presented in the next section are based on the 2008 calibration, while we report the ones based on the 2017 calibration in Section 4.4 as part of the sensitivity analysis.

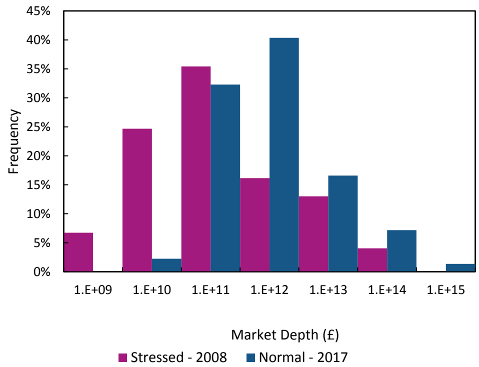


Figure 3: Market depths distribution for the 2017 and 2008 calibrations.

For government bonds we have retrieved aggregate statistics published by the relevant national authorities on volumes traded and prices from S&P indices (see Section 7.2.3 of the Appendix for the data sources). Table 2 reports for selected countries the estimated market depth of government bonds, under the 2017 and 2008 calibrations, and the corresponding price impacts that a forced liquidation of 1 bn pound would generate. We proceeded similarly for equities, using the available public data (see Section 7.2.5). As there is no public data for corporate bonds trading volumes, we relied on outstanding amounts to estimate them. Further details of the calibration and data sources are reported in the Appendix 7.2.2.

[21] To do so we have scaled down the market depths obtained under the 2017 calibration. While we are agnostic on the source of market stress, equation 16 shows that scaling market depths can be done by either scaling the trading volumes, market volatility, or the time horizon over which banks sell assets.

Table 2: Average daily volumes, market depths and associated price impacts for government bonds for selected countries.

Country	ADV (£bn)	Market depth (£bn)		Price impact of £1bn sale (bp)	
	2017	2017	2008	2017	2008
US	378	125,650	43,395	0.1	0.2
China	32	13,548	4,679	0.7	2.1
Japan	33	12,474	4,308	0.8	2.3
Canada	18	5,240	1,810	1.9	5.5
Spain	19	4,712	1,627	2.1	6.2
Germany	14	4,069	1,405	2.5	7.1
UK	26	3,513	1,213	2.9	8.2
Korea	4.8	2,006	693	5.0	14
France	7.8	1,992	688	5.0	15
Italy	4.2	1,034	357	9.7	28
Belgium	2.1	455	157	22	64
Netherlands	1.4	377	130	27	77
Sweden	1.0	336	116	30	86
Hong Kong	1.0	257	89	39	11
Singapore	0.9	226	78	44	13
Finland	0.4	138	48	72	21

4 Empirical Results

4.1 Solvency shocks

We first consider fire sales following a shock to banks' solvency. We anchor the solvency shock to the annual cyclical scenario (ACS) of the Bank of England's 2017 stress test. The scenario, which is more severe than the financial crisis, incorporates deep simultaneous recessions in the UK and global economies, large falls in asset prices and a separate stress of misconduct costs.[22] This scenario, after accounting for banks' management actions in response to the stress, caused an aggregate decrease in banks' risk-weighted capital ratios of 5.1 percentage points and an aggregate decrease in banks' leverage ratios of 1.1 percentage points (Bank of England, 2017a). Banks' low-point capital ratios are reported in Table 3.

A bank's performance in the Bank of England stress test is assessed against a hurdle rate, given by the sum of the internationally agreed minimum standards for risk-weighted capital and leverage ratios, plus any uplift set by the Bank of England. Globally systemically important banks (G-SIIs) are held against higher standards - the 'systemic reference point'. In line with the Bank of England stress test framework,

[22]See Bank of England (2017a) for more details.

we take the systemic reference point (as shown in Table 3) as the minimum capital standard banks must maintain in our model.

Table 3: Banks' capital and leverage ratios in 2017 stress test.

	Risk-weighted capital ratio (%)		Leverage ratio (%)	
	Low point	Systemic ref. point	Low point	Systemic ref. point
Barclays	7.4	7.9	3.6	3.6
HSBC	8.9	8.0	4.5	3.7
LBG	7.9	7.5	3.9	3.3
Nationwide	12.3	8.4	4.5	3.3
RBS	7.0	7.4	4.0	3.5
San UK	9.7	7.6	3.3	3.3
SCB	7.6	7.0	4.7	3.4

Source: Bank of England (2017a)

Table 4: Aggregate losses by solvency shock.

Scenario	Aggregate losses (£bn)	Losses (% CET1 capital)
Baseline	61	25
+20%	73	30
+40%	85	34
+60%	98	39
+80%	110	44
+100%	122	49

As our baseline solvency shock, we apply the losses banks incurred in the stress scenario, and assume all these losses were incured on non-tradable assets. Any banks that fall below their regulatory requirements respond to the stress by selling assets. We then consider higher intensity versions of this shock, by scaling up the initial losses suffered by each bank in increments of 20%. The aggregate losses in each scenario are given in Table 4.

Figure 4 shows the aggregate banking system losses (as a percentage of aggregate equity) due to a first round of fire sales for the different severities of the solvency shock.[23] To draw out the drivers of the results, we run the model when banks face the full set of regulatory constraints and when they are subject to (a) the leverage ratio only and (b) the risk-weighted capital ratio only.[24]

[23] We only show results for one round of sales as losses in subsequent rounds are generally insubstantial.

[24] The LCR is unaffected by losses on non-marketable assets, so would not trigger any fire sales and is not shown in the figure.

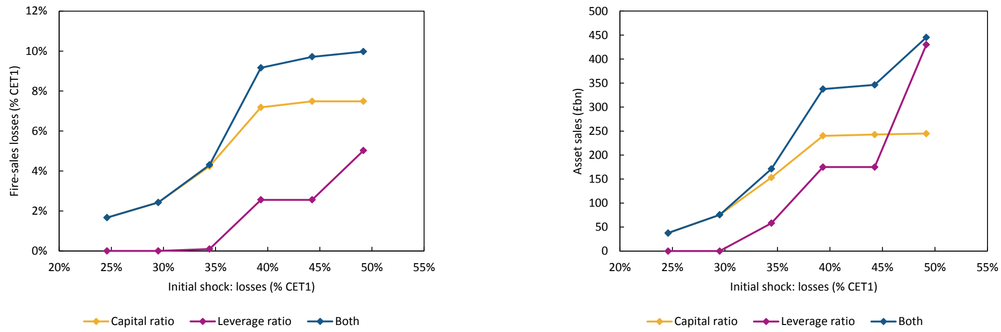


Figure 4: Aggregate losses (left) and sales (right) following solvency shocks.

For the smallest initial shocks, fire-sale losses are minimal, as few banks breach their regulatory capital requirements and those that do are able to restore their regulatory ratios with minimal losses. As the shock increases in intensity, more banks are forced into fire sales and losses increase, reaching around 10% of initial equity for the most severe shock. Losses appear to be mainly driven by the risk-weighted capital constraint, which generates larger losses than the leverage constraint. As we explain below, this is because banks sell larger quantities and less liquid assets when constrained by the capital ratio requirement.

For most shocks the risk-weighted capital ratio causes banks to sell more assets than the leverage ratio, as shown in the right-hand panel of Figure 4. This is for two reasons. Firstly, the baseline solvency shock we have adopted affects firms' risk-weighted capital more than their leverage ratio. This is driven by the choice of the scenario and banks' balance sheet compositions. Secondly, and more generally, cash holdings count towards the leverage ratio but not towards the risk-weighed capital ratio, as cash has a risk weight of zero. Thus when banks breach their capital ratio, they are immediately forced to sell assets to restore their ratios, while when they breach their leverage ratio they can first use their cash to deleverage and thus avoid selling assets.

As shocks increase in intensity, the leverage ratio begins to play a larger role, leading to larger sales of assets for the largest shocks. This is because the set of assets whose sale could boost a bank's risk-weighted capital ratio is smaller than the set whose sale would boost a firm's leverage ratio, as explained in Proposition 1 and Figure 1. For example, banks would never sell zero-risk-weight government bonds to improve their risk-weighted capital ratio, but would to boost their leverage ratio.

Furthermore, the risk-weighted capital ratio incentivises banks to sell less liquid assets on average, as shown in Figure 5, which plots the average price impact of sales under different constraints. A bank subject to a leverage requirement will choose which assets to sell solely based on the losses this would cause: it would first sell assets which have a more liquid secondary market and that it does not hold in large quantities. A bank faced with a risk-weighted capital ratio must take into account assets' risk weights as well as their liquidity. This tends to lead it to sell less liquid assets than if it only faced a leverage constraint, which leads to greater price impacts.

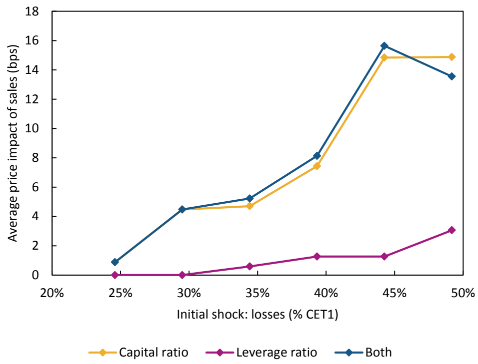


Figure 5: Average price impact of sales following solvency shocks.

More specifically, as set out in Section 2.5, selling a unit of asset k results in losses equal to the ratio of a bank's holdings of k to the asset's market depth, m_k/δ_k. Thus a loss-minimizing bank would prefer to sell assets in descending order of the ratio δ_k/m_k. The left panel of Figure 6 takes a given bank in a given shock scenario, subject only to a leverage ratio requirement, ranks its assets by the ratio of market depth to holdings, and plots this rank against the percentage sold. In line with the analytical result[25] of Section 2.5, the bank does indeed sell assets in descending order of the ratio δ_k/m_k.

The right panel of Figure 6 takes a bank subject to only the risk-weighted capital ratio requirement, and again plots percentage sold against the same ranking of assets. This bank does not follow the same strategy: in particular, it sells off several assets which it holds in large amounts and are illiquid. These sales cause it large losses. As explained in Section 2.5, it deviates from the loss-minimizing strategy due to risk weights: when faced with a binding risk-weighted capital ratio it sells assets in descending order of the ratio $\rho_k(1 + \delta_k/m_k)$, as shown in Figure 7.

[25] Note that the numerical solution need not always exactly match the analytical solution, as the assumptions of Propositions 2 and 3 might not hold. We compare the numerical and analytical solutions in the Appendix, and find that the the two are typically very close.

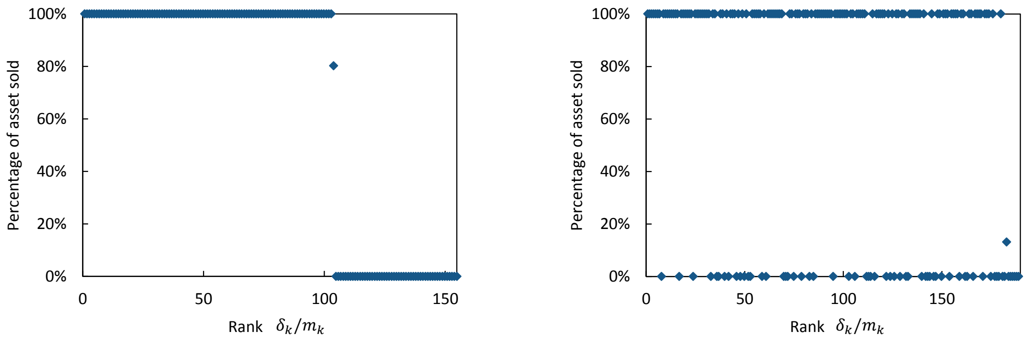


Figure 6: Sales ordered by assets' market depth to holdings size ratio: under the leverage ratio (left) and the risk-weighted capital ratio (right) constraint.

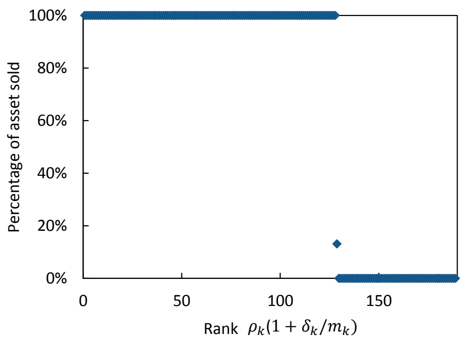


Figure 7: Sales ordered by assets' risk-weighted market depth to holdings size ratio under the risk-weighted capital constraint.

As a result of these different liquidation strategies, we find that for the same shock size banks sell a larger proportion of government bonds under the leverage ratio relative to the capital ratio as shown in Figure (8). Under the capital ratio banks tend to sell a large amount of equities, as these are very liquid and have high risk weights.

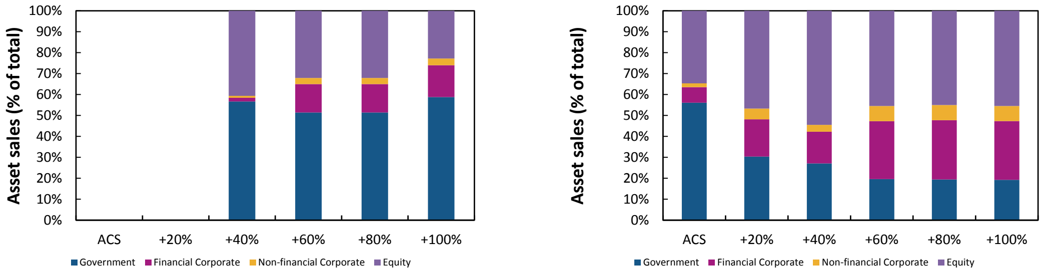


Figure 8: Sales by asset class: under the leverage ratio (left) and the risk-weighted capital ratio (right) constraint.

4.2 Funding shocks

We base our funding shock on the outflows stress in the LCR regulation. The LCR assumes outflows over a 30 day stress across different liability classes, ranging from 100% outflows for certain types of short-term wholesale funding, to 5% for stable deposits and 0% for debt with a maturity greater than one month.[26] As our baseline shock we assume these outflows are realised, which amounts to around 12% of liabilities.[27] To vary the severity of the liquidity shock we scale these outflows up and down by increments of 20%, as shown in Table 5.[28]

Table 5: Aggregate outflows in variants of the Liquidity Coverage Ratio scenario

Scenario	Outflows (£bn)	Outflows (% balance sheet)
−60%	258	5
−40%	387	7
−20%	516	10
LCR	645	12
+20%	774	15
+40%	903	17
+60%	1032	20
+80%	1160	22

[26]The LCR scenario assumes inflows as well as outflows, but we exclude these from our scenario.

[27]By comparison, Northern Rock saw a 30% decrease in its liabilities in the final six months of 2007, with retail and unsecured wholesale funding each falling by over 50% (see Shin (2009)).

[28]Whenever our scaling of the shock implies outflows exceed 100% of a given liability, we cap the outflows at 100% and assume the residual outflows come from the remaining liabilities.

Following the shock, banks are faced with outflows that need to be paid out in cash, and potentially an LCR below 100%. Figure 9 shows sales and resulting losses for different funding shock sizes, assuming banks aim to defend their LCR at 100%.

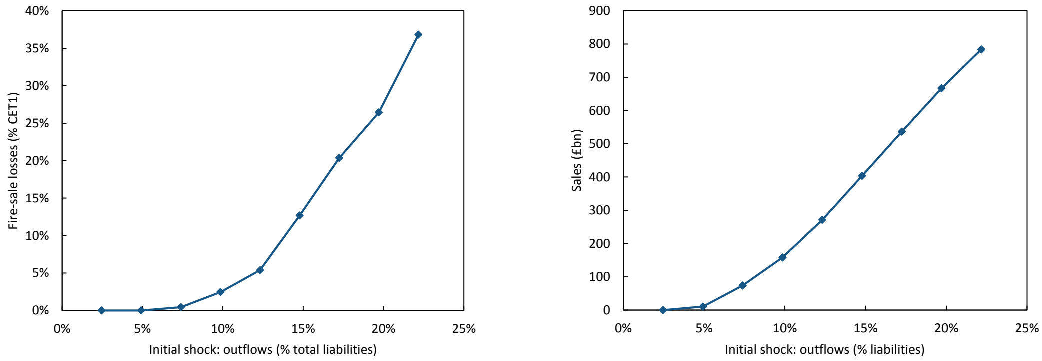


Figure 9: Aggregate losses (left) and sales (right) following funding shocks when banks defend their LCR above 100%.

For small initial outflows, fire-sale losses are minimal. This is because most banks have significant headroom over an LCR of 100% and have large holdings of cash-like assets, and so can avoid selling assets by using their initial cash to meet creditor withdrawals. As the shock increases, banks run out of headroom and are forced into liquidating assets. Sales lead to losses ranging from 5% of aggregate CET1 capital for the initial outflows envisaged in the LCR regulation up to 37% of aggregate capital for the largest shock,[29] which assumes initial outflows of 22%.

Funding shocks have the capacity to cause much larger fire-sale losses than solvency shocks. This reflects which assets banks find it optimal to liquidate when hit by the different shocks. When banks are selling assets in order to restore their capital or leverage ratios, they have an incentive to sell an asset only if doing so will actually improve these ratios. Where a bank holds sufficient amounts of a sufficiently illiquid asset, selling this asset will cause large enough losses that the bank's solvency will be harmed, rather than helped, by the sale. As such, banks avoid selling the assets that impose the largest losses on themselves (Proposition 1). In doing so, they also avoid imposing large losses on other holders of these illiquid assets.

By contrast, when faced with large enough funding withdrawals, a bank's main concern is simply raising cash. Selling *any* asset will raise cash - even highly illiquid ones. Whilst banks will seek to minimise losses where possible when raising funds, for the largest shock they will find it optimal to sell all assets rather than failing to pay out their creditors. Thus the set of assets banks are willing to sell in response

[29]For this shock all banks fail to restore their LCR to 100% regardless of how much they sell, hence we have not increased the shock further.

to funding shocks is greater than the set of assets they are willing to sell in response to solvency shocks, and includes assets that impose large losses on themselves and others. The result of this is that the upper bound on fire-sale losses following a funding shock, in our model, is almost four times as large as the upper bound on losses following a solvency shock.

The LCR was designed in order to ensure banks have sufficient liquid assets to meet net outflows over a stress of 30 days. Whilst banks are required to exceed 100% LCR in normal times, the liquid asset buffer is intended to be usable, and so can be allowed to fall below 100% in a stress. Nevertheless, banks may in certain circumstances be reluctant to use some or all of their liquid asset buffers, for example for precautionary reasons to leave liquidity in case of further shocks. We therefore run the model to analyse fire sales when banks are willing to use their liquid asset buffers to varying degrees - these scenarios range from banks using all their liquid assets to situations where they do not allow their LCR to fall below 100%.

Figure 10 shows aggregate losses for different severities of the shock, assuming banks defend their LCR above different thresholds. The magnitude of losses depends on what liquidity position banks defend. For initial outflows of 12% of liabilities, the losses when banks defend an LCR of 100% are roughly three times bigger than if they allow their LCR to fall to 75% or below. For larger shocks, however, the outflows are so large that they need to sell a large proportion of their assets regardless of whether they defend their LCR at 100% or not.

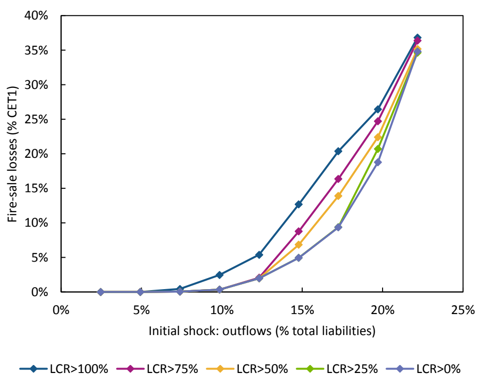


Figure 10: Aggregate losses following funding shocks when banks defend different levels of the LCR.

Banks adopt different liquidation strategies, depending on whether they see the LCR as a binding constraint or not. As discussed in section 2.5, when banks aim to defend their LCR they will need to sell assets with low impact on the ratio, in particular assets that do not cause large falls in HQLA or expected inflows. If instead banks are willing to let their LCR fall below 100%, they are more able to use cash and highly liquid assets to pay out their creditors, and thus avoid selling assets that cause them large losses. As a result the average liquidity of assets sold is higher when banks defend their LCR relative to when they are willing to use their liquid buffers, as shown in Figure 11.

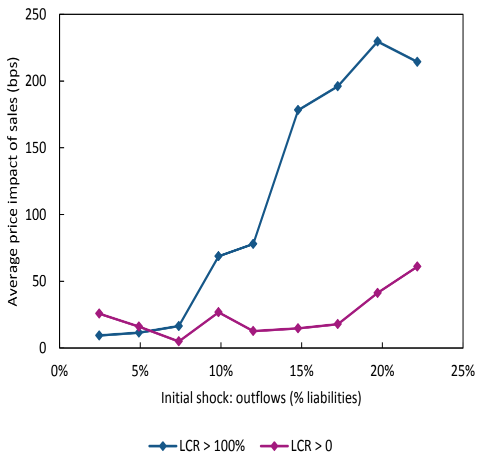


Figure 11: Average price impact of sales following funding shocks when banks defend their LCR above 100% and when they do not.

Figure 12 shows the assets sold by asset class. The liquidation strategies are fairly homogeneous, with banks selling equities (for small shocks) and government bonds (for larger shocks) in larger proportions than corporate bonds. However, for the same initial outflows, banks sell larger proportions of government bonds when they are willing to draw down their LCR rather then defend it above 100%.

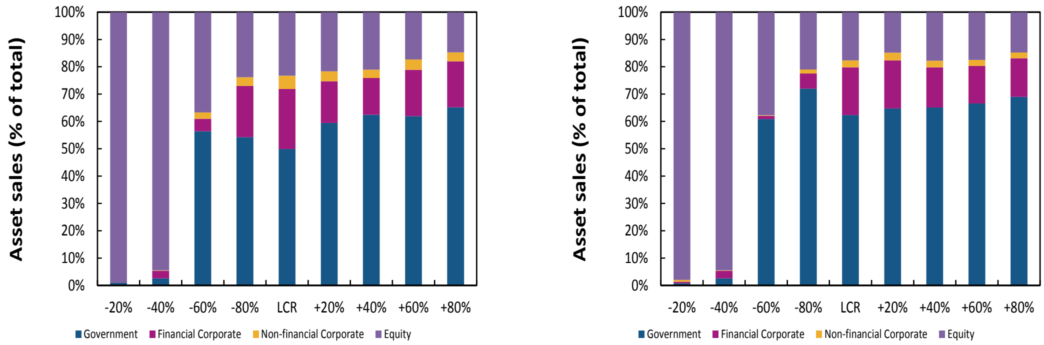


Figure 12: Sales by asset class: when banks defend their LCR above 100% (left) and when they do not (right).

4.3 Combined funding and solvency shock

In reality, shocks to the economy have implications for both solvency and liquidity. The 2008 crisis saw banks face large losses on assets such as mortgage-backed securities at the same time as facing funding stresses as their creditors withdrew funds (Gorton and Metrick, 2012; Hellwig, 2009). To assess the potential for fire sales in these more general shocks, we run combinations of the solvency and funding shocks described in the previous two sections.[30] In this case, both solvency and liquidity requirements are relevant.

Figure 13 summarises our findings. Losses are increasing in both initial losses and initial outflows. However, consistent with the results for the shocks in isolation, losses rise more sharply for increased funding shocks than for solvency shocks, and the maximum losses under pure solvency shocks are significantly smaller than for pure liquidity shocks.

Composite funding and solvency shocks result in fire-sale losses that tend to be less than the sum of the losses following the equivalent funding and solvency shocks in isolation (left panel of Figure 14). This difference is relatively small for the smallest shocks, but exceeds 10% of equity for the largest composite shocks. This is because the assets banks sell in response to a solvency shock will often also help them in response to a liquidity shock, and vice versa (see Figure 1). Selling an asset in order to pay out running depositors will - provided losses are not too great - also boost a bank's leverage ratio. Furthermore, if this asset does not have a zero risk weight, selling it will also often boost a bank's risk-weighted capital ratio. As such, banks' responses to solvency and liquidity shocks are complementary and, in terms of fire-sale losses, a composite shock is often less than the sum of its individual parts.

There are, however, interactions between solvency and funding shocks that mean a composite shock results in greater losses than either a funding shock or a solvency shock in isolation. The right panel of Figure 14 plots the difference between losses following the combined shock and the larger of the losses following a funding or solvency shock in isolation. These excess losses are positive for two reasons. Firstly, some banks are more vulnerable to solvency shocks, while other banks are more vulnerable to funding shocks. A composite shock means both these sets of banks are forced into asset sales, resulting in larger aggregate sales and thus higher losses. Secondly, whilst there is substantial overlap between the sets of assets that help banks respond to funding and solvency shocks, there are a number of assets whose sale would not improve a bank's solvency but would help it raise cash to pay out depositors. Thus if a bank has exhausted all of the assets that improve its solvency in response to a solvency shock, adding a liquidity shock will cause it to sell more assets. And, by definition, these assets are those that cause large losses to the bank. As a result, the

[30]Note that the two shocks are applied simultaneously in this model, while in reality funding and solvency shocks might play out dynamically, for instance with funding runs following solvency concerns.

excess losses for large solvency and moderate funding shocks can be extremely large. For the largest shocks the excess losses banks incur are small, as they are forced to sell almost all their assets in response to the funding shock in isolation.

In general, these results highlight that it is important to consider solvency and funding shocks together, accounting for their interaction. Running the two stress scenarios independently and either considering the results in isolation or aggregating them would lead to an under- or over-estimation of fire-sale losses respectively.

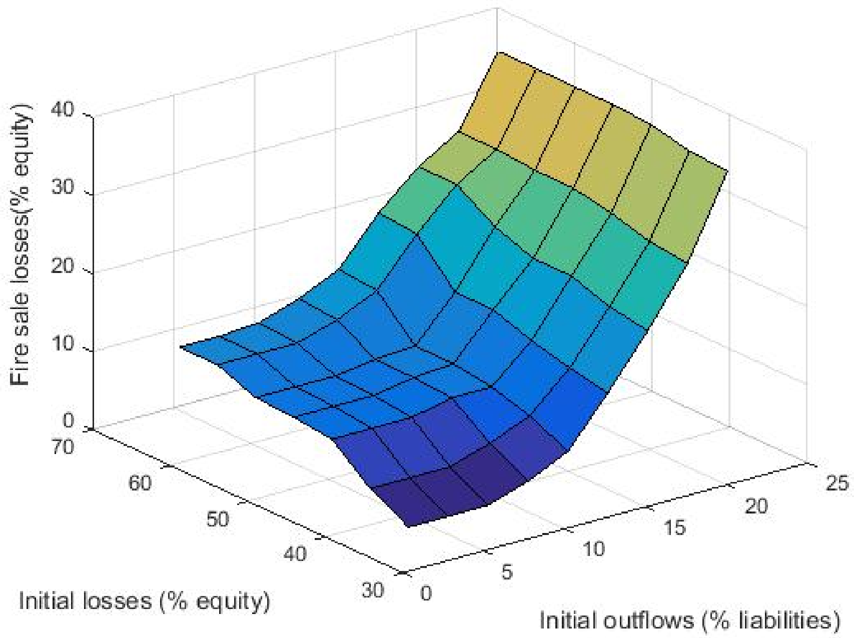


Figure 13: Fire-sale losses for composite solvency and funding shocks.

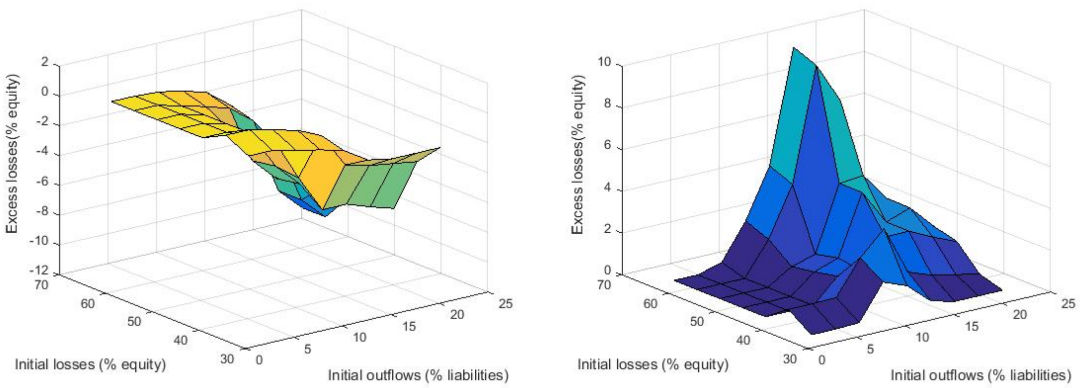


Figure 14: Losses for composite solvency and funding shocks in excess of the sum of the individual shocks (left) and in excess of the largest of the individual shocks (right).

4.4 Additional analyses

In this section we first show how the results of the model change when we vary the market depths. We then analyse the effects of implementing the UK framework for the leverage ratio rather than the international standard. Finally we compare the fire-sale losses generated in our model when banks sell assets optimally with the losses when they sell assets proportionally, as is commonly assumed in the literature.

Market depth and price impact. Price impacts are hard to measure, but play a fundamental role in determining the extent of fire-sale losses. The results presented in the previous section are based on market depths calibrated using 2008 data, i.e. under stressed market conditions. In this section we show the key results using market depths calibrated using 2017 data, i.e. under normal market conditions.

Markets were more liquid in 2017 relative to 2008, with the median market depth across all tradable assets in 2017 implying a drop in prices of 0.6% (under a linear price impact) for a forced liquidation of 1 bn GBP over 5 days, and the median market depth in 2008 implying a 4% decrease. Therefore, we expect fire-sale losses to be smaller based on this calibration. This is indeed the case, as shown in Figure 15. The amounts of assets sold are relatively similar in the two calibrations, but losses are up to three times as large when market depths are calibrated under stressed conditions. Similar results are obtained for the funding shock scenarios.

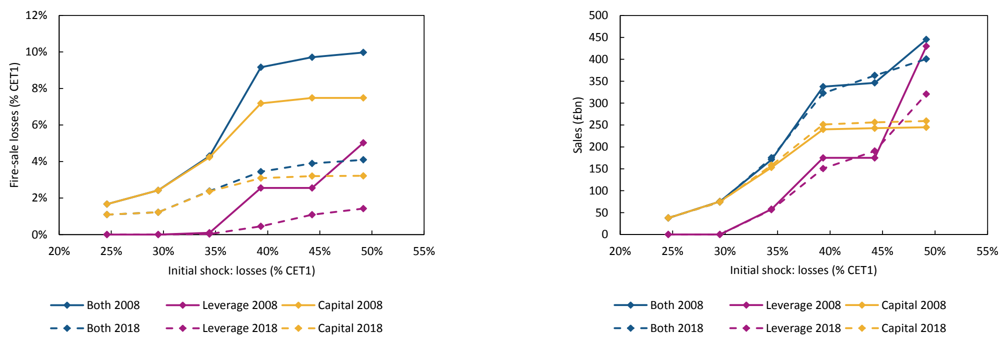


Figure 15: Aggregate losses (left) and sales (right) following solvency shocks using stressed and normal market depths

The UK leverage ratio. The UK leverage framework differs from the international standard. Specifically, the UK leverage ratio framework excludes from the calculation of the total exposure measure those assets constituting claims on central banks, where they are matched by deposits accepted by the firm that are denominated in the same currency and of identical or longer maturity.[31] To ensure that this would not result in higher banks' leverage, the Bank of England has also increased the minimum leverage ratio requirement to 3.25%.

In the rest of the paper we have followed the international standards, whereby what we define as the cash asset bucket, which includes claims on central banks, does contribute to leverage exposures. In this section we investigate how implementing the UK leverage framework affects the results of the model. Under the international leverage framework, cash provides a buffer against fire sales: even after a shock forces a bank below its minimum requirements, it is not forced to sell assets if it has cash available to deleverage. Under the UK leverage framework, in our model, once a bank falls below its leverage requirement it is forced to sell assets, as depleting central bank reserves will not help. As a result, solvency shocks might lead to fire sales under the UK leverage framework but not under the international framework.[32] Figure 16 demonstrates this result: banks subject to the UK leverage ratio face larger losses for a given shock, owing to the fact they have to sell more assets. Thus the UK's modification to the leverage ratio might have the unintended consequence of increasing fire-sale losses in certain scenarios.

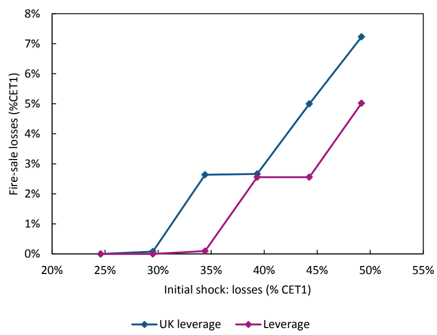


Figure 16: Aggregate losses under the leverage ratio following solvency shocks when banks are subject to the UK or the international leverage framework.

[31]Central bank claims for these purposes include reserves held by a firm at the central bank, banknotes and coins constituting legal currency in the jurisdiction of the central bank, and assets representing debt claims on the central bank with a maturity of no longer than three months. See `https://www.bankofengland.co.uk/-/media/boe/files/prudential-regulation/policy-statement/2017/ps2117.pdf` for more details on the UK leverage ratio framework.

[32]Note that, since banks collectively have to hold the amount of reserves that the central bank creates, in a *systemic* shock the banking system cannot deleverage by depleting central bank reserves. Therefore, whether they were included in the calculation of the leverage ratio would not affect the results in aggregate. Nevertheless, bank-specific losses would be affected.

Optimal vs proportional deleveraging. A key contribution of our paper is to allow banks to optimise over which and how many assets they sell. Previous work typically assumes that banks simply sell assets in proportion to their initial holdings of the assets (Cont and Schaanning, 2017; Greenwood et al., 2015). This assumption has a large impact on the the magnitude of losses in a fire sale. The left panel of Figure 17 shows the results of the solvency shock for when banks sell off assets optimally, as in Section 2, vs when they sell off assets proportionally. To ensure comparability with Cont and Schaanning (2017) and Greenwood et al. (2015) we assume banks only face a leverage ratio constraint. The losses under proportional deleveraging are over 5 times larger for the most severe shocks. The right panel of Figure 17 shows why: under a proportional selling rule the assets banks sell are significantly less liquid, as when banks optimise they avoid selling assets that will cause them large losses. Papers with proportional selling are therefore likely to significantly overstate losses in a fire sale.

Allowing banks to optimise over which assets they sell has implications for *where* one would expect fire-sale losses to occur. Figure 18 breaks up the losses banks incur in the fire sale by the liquidity of the assets in which banks incur them. With optimal deleveraging, the vast majority of fire-sale losses are incurred in the most liquid asset classes. This is due to the portfolio overlap being largest in these assets, as well as due to banks avoiding the sale of illiquid assets to minimise liquidation losses. With proportional deleveraging, banks no longer avoid selling illiquid assets, and the losses are spread out more evenly across different assets. Thus whilst previous models might lead to concerns about fire-sale losses on banks' illiquid assets, allowing for more rational behaviour by banks suggests the most liquid assets are actually where banks are most likely to sustain material losses, when the leverage ratio is the binding constraint.

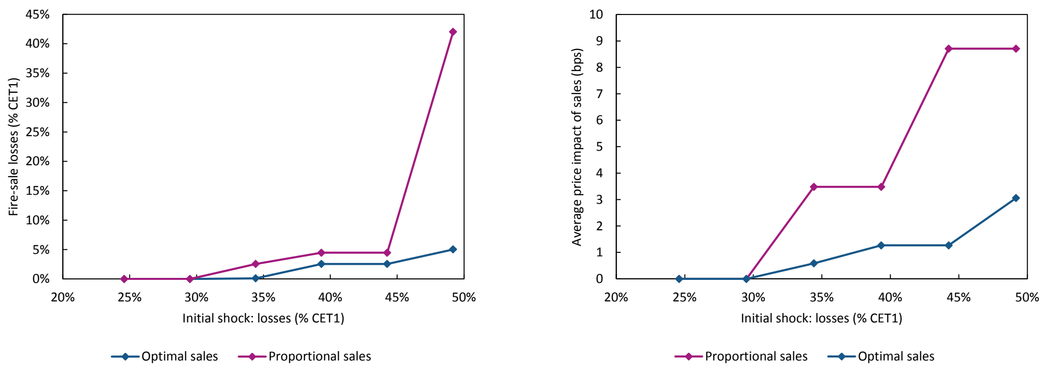


Figure 17: Losses (left) and average price impact of sales (right) for solvency shocks under proportional and optimal deleveraging.

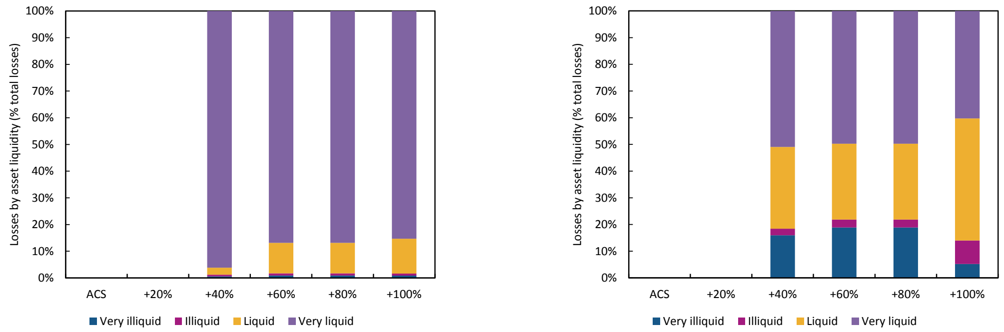


Figure 18: Loss by quartile of asset liquidity for optimal (left) and proportional (right) asset sales following solvency shocks

5 Potential extensions

While the model aims to provide a realistic description of how bank fire sales might occur, it has several caveats that are important to keep in mind when interpreting the results.

The calibration of market depths is a challenging task. Changes in sample periods and data granularity can lead to quite different price impacts. We do have instrument-level data on bank holdings available so an interesting extension of the model could be to run it at the instrument level. The choice of the level of aggregation has ambiguous effects on the contagion channel and fire-sale losses. On the one hand, using more coarse data will overstate common holdings between firms. On the other hand, by combining markets, market depth will increase and so the price impact of given sales will be smaller.

We have adopted a linear price function to make the problem more tractable. Although this functional form is likely to be a fairly good approximation for small sales, it might overstate the losses for large sales. Alternative price functions have been developed and used in other work, such as the exponential price function (used for instance in Cont and Schaanning (2017)). This function can generate more realistic dynamics, as the impact decays as the trade size increases, which is consistent with there being "value" investors in the market who will step in to buy the asset if the price falls sufficiently.

The impact of asset sales by banks will ultimately depend on the behaviour of other market participants and so is intrinsically uncertain. We only focus on seven UK banks' selling behaviour, abstracting from the buy side and the non-banking sector. Only considering the seven stress-test banks might lead to an underestimation of contagion if other institutions holding the same assets were also to sell assets in the scenarios we consider. However, other institutions could also dampen fire-sale losses by taking the demand side and acting as stabilisers.

We have excluded derivatives from our definition of marketable assets and made a neutral assumption on the impact of interest rate swap markets. Government bonds held as part of a liquid asset buffer may be hedged with interest rate swaps. If they are, then sales of these assets would likely be done alongside an unwinding of the pay-fixed interest rate swaps, and so as the bond yield rises, the swap rate may fall, leading to further losses. However if the assets are hedged with a broader set of banking book assets, or if they are held on the trading book, then sales might not be accompanied by unwinding of swap positions and so no extra losses would occur. More detailed studies using granular balance sheet data are needed to shed light on this.

Finally, we currently ignore important dynamic effects that are worth exploring in the future. For example, it would be interesting to quantify the amount of funding that banks lose as a function of the loss to their assets' values (see Pierret (2015)), which would allow us to have a complete feedback loop from solvency to liquidity and further increase the realism of the described mechanisms.

6 Conclusions

The paper develops a flexible and realistic model of fire sales that reflects the post-crisis regulatory environment and can be operationalized in stress testing models. Most existing models of fire sales are too stylised for direct use in policy analysis or to study contagion risk in a realistic setting.

We fill this gap by developing a quantitative model of fire sales, in which banks are constrained by risk-weighted capital and leverage regulation, as well as liquidity regulation should banks aim to defend a certain level of the LCR. Banks in our model minimise the losses they incur due to their own sales when they deleverage, instead of following some heuristic deleveraging scheme. We apply the model to the seven UK banks subject to the regulatory stress test in 2017. The framework enables us to explore the likelihood, causes and magnitude of fire sales in different stress scenarios.

Following solvency shocks, risk-based capital requirements tend to be more tightly binding and incentivise banks to sell larger amounts of illiquid assets relative to the leverage ratio, which in turn leads to larger fire-sale losses. Nevertheless, fire-sale losses due to solvency shocks remain moderate even for severe shocks. In contrast, severe funding shocks can lead to large fire-sale losses. Thus models that focus on solvency shocks and only include a leverage ratio may be missing two key drivers of fire-sale losses.

Existing quantitative models of fire sales tend to assume that banks sell off assets in proportion to their initial holdings. In reality, banks are unlikely to follow this strategy, as it involves them taking actions that cause themselves significant losses. Allowing banks to optimise their liquidation strategy results in significantly lower losses than assuming that they sell assets proportionally to holdings. This also implies that the assets more likely to transmit losses in a fire sale are liquid, rather than illiquid, assets.

Finally the model delivers some interesting implications for policy. First, we demonstrate the importance of ensuring that liquidity buffers are usable in stress. If banks aim to defend their liquidity positions by protecting their liquid asset buffers, they may fire sell illiquid assets, resulting in large losses. If, as emphasized by regulators, they fully utilise their liquid asset buffers then losses can be reduced. Second, we highlight a potentially negative side-effect of the new UK leverage framework, which excludes central bank reserves from the leverage ratio. Doing so removes a bank's ability to use central bank reserves to deleverage once they breach their leverage ratio requirements, potentially leading to larger losses in a fire sale.

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7 Appendix

7.1 Proofs

7.1.1 Proof of Proposition 1

To ease notation, we drop the index i refering to a specific institution in the proofs. Thus, the (column-)vector M_i is denoted by m, S_i by s, e_i by e and O_i by o.

Leverage constraint. Substituting the cash constraint (15) into the leverage constraint (13) gives:

$$\beta^{LEV} \leq \frac{e - m^\top D^{-1} s}{(m-s)^\top(\mathbf{1} - D^{-1}s) + \mathbf{1}^\top o}.$$

Rearranging gives:

$$-\beta^{LEV} s^\top \mathbf{1} + \beta^{LEV} s^\top D^{-1} s - \beta^{LEV} m^\top D^{-1} s + m^\top D^{-1} s \leq e - \beta(m^\top + \mathbf{1}^\top o) \quad (17)$$

Fix sales of all assets except asset k. The difference between the left-hand side of equation 17 for general s_k vs $s_k = 0$ is given by:

$$\frac{m_k s_k}{\delta_k} - \beta^{LEV}(s_k + \frac{m_k s_k}{\delta_k} - \frac{s_k^2}{\delta_k})$$

Note that the first term is the losses due to the sale, whilst the term in brackets is the reduction in assets due to the sale. Setting $s_k > 0$ will never ease a bank's leverage constraint if this figure is positive for all $s_k \in (0, m_k]$. This is true if:

$$\frac{m_k}{\delta_k} > \frac{\beta^{LEV}}{1 - \beta^{LEV}} \quad (18)$$

Risk-weighted capital constraint. The risk-weighted capital ratio constraint is:

$$\beta^{CAP} \leq \frac{e - m^\top D^{-1} s}{\rho_M^\top\left[(m-s) \circ (\mathbf{1} - D^{-1}s)\right] + \rho_O^\top O},$$

Following the same steps as above, we derive the following: setting $s_k > 0$ will never ease a bank's risk-weighted capital constraint if the following is positive for all $s_k \in (0, m_k]$:

$$\frac{m_k s_k}{\delta_k} - \beta^{CAP} \rho_k^M (s_k + \frac{m_k s_k}{\delta_k} - \frac{s_k^2}{\delta_k})$$

The first term is again the losses due to the sale, whilst the term second is the reduction in risk-weighted assets, multiplied by the capital requirement. This condition holds if:

$$\frac{m_k}{\delta_k} > \frac{\rho_k^M \beta^{CAP}}{1 - \rho_k^M \beta^{CAP}}$$

LCR constraint. The LCR constraint is:

$$\beta^{LCR} \leq \frac{\lambda^\top [(m-s) \circ (\mathbf{1} - D^{-1}s)] + c + s^\top(\mathbf{1} - D^{-1}s) - \mathbf{1}^\top R}{\omega_{out}^\top (L-R) - \omega_{in}^\top [(m-s) \circ (\mathbf{1} - D^{-1}s)]}.$$

Following the same steps as above yields that it will never help a bank ease its LCR constraint to sell asset k if for all $s_k \in (0, m_k]$:

$$s_k(1 - \frac{s_k}{\delta_k}) - (\lambda_k + \omega_k^{in}\beta^{LCR})(s_k + \frac{s_k m_k}{\delta_k} - \frac{s_k^2}{\delta_k}) < 0$$

The first term is the cash raised from the sale, whilst the second is the reduction in assets due to the sale, weighted by the asset's liquidity weight plus the bank's LCR target multiplied by the asset's inflow rate. Rearranging as above yields that a bank should never sell asset k if:

$$\frac{m_k}{\delta_k} > \frac{1 - (\lambda_k + \omega_k^{in}\beta^{LCR})}{(\lambda_k + \omega_k^{in}\beta^{LCR})}$$

7.1.2 The Fractional Knapsack Problem.

Let $p, w \in \mathbb{R}^n_{++}, a \in \mathbb{R}$. The Fractional Knapsack problem is: Solve

$$\max \sum_{i=1}^{n} p_i x_i \tag{19}$$

$$\text{subject to} \sum_{i=1}^{n} w_i x_i \leq a \tag{20}$$

$$0 \leq x_i \leq 1. \tag{21}$$

Denote by x^* the optimal solution. Suppose WLOG $\frac{p_1}{w_1} \geq \frac{p_2}{w_2} \geq \cdots \geq \frac{p_n}{w_n}$. Then $x^* = \cdots \geq x^*_{k-1} = 1 \geq x^*_k \geq 0 = x_{k+1} = \cdots x_n$, and $a - \sum_{i=1}^{k-1} w_i \leq w_k, x^*_k = \frac{a - \sum_{i=1}^{k-1} w_i}{w_k}$.
Proof. See standard references, e.g. Prop 17.1 and Thm 17.3 in (Korte and Vygen, 2012).

Remark. $w_i \leq 0$ implies $x^*_i = 1$. Indeed, suppose $x^*_i < 1$. Then $y_j = x^*_j \forall j \neq i, y_i = 1$ satisfies $\sum_k w_k y_k = \sum_k w_k x^*_k + w_i(1 - x^*_i) \leq a$. And $\sum_{k=1}^n p_k x^*_k = \sum_{j=1}^n p_j y_j - p_i y_i + p_i x^*_i = \sum_{j=1}^n p_j y_j + p_i(x^*_i - 1) < \sum_{j=1}^n p_j y_j$.

7.1.3 Leverage Ratio

Proof. Proof of Proposition 2. Substituting the cash constraint (15) into the leverage constraint (13) gives:

$$\frac{e - m^\top p}{(m-s)^\top(\mathbf{1} - D^{-1}s) + o} \geq \beta,$$

where (for aesthetic reasons) we write β for β_i^{LEV}, the leverage requirement. Rearranging the constraint gives

$$e - m^\top D^{-1}s - \beta m^\top(\mathbf{1} - D^{-1}s) + \beta s^\top \mathbf{1} - \beta o - \beta s^\top D^{-1}s \geq 0.$$

Call the left-hand side of this inequality $g(s)$ for $g : \mathbb{R}^{K_M} \to \mathbb{R}$. The first-order Taylor approximation of this function at $s = (0, ..., 0)^\top$ is given by

$$g(s) = e - \beta m^\top \mathbf{1} - \beta o + \big((\beta - 1)m^\top D^{-1} + \beta\big)s + \mathcal{O}(s^2).$$

The constraint, which was quadratic previously, is now linear:

$$e - \beta m^\top \mathbf{1} - \beta o + \big((\beta - 1)m^\top D^{-1} + \beta\big)s \geq 0.$$

Denote $x := (x_1, ..., x_{K_M})^\top$ such that $s = (\mathbf{1} - x) \circ m$ means x_k is the proportion of asset k *that is kept*. With this change of variable, the problem has the exact form of the standard continuous Knapsack problem, as in (19) – (21) with

$$\begin{aligned} p_k &:= \frac{m_k^2}{\delta_k} \\ w_k &:= \Big((\beta-1)\frac{m_k}{\delta_k} + \beta\Big)m_k \\ a &:= e - \beta m^\top \mathbf{1} - \beta o + \big((\beta - 1)m^\top D^{-1} + \beta\big)m. \end{aligned}$$

Hence, the solution is to *keep* assets in descending order of the ratio

$$\frac{p_k}{w_k} = \frac{\frac{m_k}{\delta_k}}{(\beta-1)\frac{m_k}{\delta_k} + \beta},$$

which is equivalent to selling the assets in descending order of the ratio $\frac{\delta_k}{m_k}$. □

7.1.4 Risk-weighted capital ratio

Proof. Proof of Proposition 3. The risk-weighted capital ratio is given by:

$$\frac{e - m^\top D^{-1}s}{\rho_M^\top \left[(m-s)\circ(\mathbf{1}-D^{-1}s)\right] + \rho_O^\top o} \geq \beta. \tag{22}$$

Rearranging, and writing β for β_i^{CAP}, gives

$$g(s) := e - m^\top D^{-1}s - \beta\rho_M^\top m \circ (\mathbf{1}-D^{-1}s) + \beta\rho_M^\top s - \beta\rho_O^\top o - \rho_M^\top \beta(s\circ D^{-1}s) \geq 0.$$

The first-order Taylor approximation of the constraint near $s = (0, ..., 0)^\top$, together with the change of variable $x_k := 1 - \frac{s_k}{m_k}$ yields the linear constraint

$$\sum_{k=1}^{K_M}(\rho_k\beta + \frac{m_k}{s_k}(\beta\rho_k - 1))x_k \leq e - \beta - \rho_O^\top o - \beta\rho_M^\top m - \sum_{k=1}^{K_M}(\rho_k\beta + \frac{m_k}{\delta_k}(\beta\rho_k - 1))m_k.$$

As in Prop. 2, this problem has thus been transformed to the standard continuous Knapsack problem with

$$\begin{aligned}
p_k &:= \frac{m_k^2}{\delta_k} \\
w_k &:= \big(\beta\rho_k + \frac{m_k}{\delta_k}(\beta\rho_k - 1)\big)m_k \\
a &:= e - \beta - \rho_O^\top o - \beta\rho_M^\top m - \sum_{k=1}^{K_M}(\rho_k\beta + \frac{m_k}{\delta_k}(\beta\rho_k - 1))m_k.
\end{aligned}$$

Consequently, assets need to be *kept* in order of decreasing ratios

$$\frac{p_k}{w_k} = \frac{\frac{m_k}{\delta_k}}{(\beta\rho_k - 1)\frac{m_k}{\delta_k} + \beta\rho_k},$$

which is equivalent to selling assets in descending order of the quantity

$$\rho_k\left(1 + \frac{\delta_k}{m_k}\right).$$

When the market depth is large relative to holdings, this quantity is close to $\frac{\rho_k\delta_k}{m_k}$. □

7.1.5 Liquidity Coverage Ratio

Proof. Proof of Proposition 4. Proceeding exactly as in the proofs of Prop. 2 and Prop. 3, and writing β for β_i^{LCR}, we can linearize the constraint near $s = 0$ to turn the problem into a standard continuous Knapsack problem with

$$
\begin{aligned}
p_k &:= \frac{m_k^2}{\delta_k} \\
w_k &:= \big(1 - (\lambda_k + \omega_k^{in}\beta)(1 + \frac{m_k}{\delta_k})\big)m_k \\
a &:= (\lambda + \omega_{in}\beta)^\top M + C + \mathbf{1}^\top R - \beta\omega_{out}^\top (L - R) + \\
&\sum_{k=1}^{K_M} \big(1 - (\lambda_k + \omega_k^{in}\beta)(1 + \frac{m_k}{\delta_k})\big)m_k.
\end{aligned}
$$

Thus banks will *keep* assets in decreasing order given by

$$\frac{p_k}{w_k} = \frac{\frac{m_k}{\delta_k}}{1 - (\lambda_k + \omega_k^{in}\beta_i^{LCR})(1 + \frac{m_k}{\delta_k})},$$

which is equivalent to selling assets in decreasing order given by

$$\frac{\delta_k}{m_k}(1 - \lambda_k - \omega_k^{in}\beta_i^{LCR}) - (\lambda_k + \omega_k^{in}\beta_i^{LCR}).$$

□

7.2 Model calibration details

We characterise banks' balance sheets using regulatory data submitted by firms for the end of 2016. In particular, we use balance sheet data collected in the Financial Reporting Framework (FINREP) under the CRD IV regulatory reporting framework.[33] We complement this dataset with data collected in the Common Reporting Framework (COREP) for the purpose of monitoring the liquidity coverage requirement, and instrument-level data on banks' tradable securities collected for the 2017 stress test.

7.2.1 Regulatory ratios

Leverage ratio. The leverage ratios are based on firms' regulatory returns. The Bank of England has chosen to remove claims on central banks from the calculation

[33] See `https://www.eba.europa.eu/documents/10180/359626/Annex+V_Instructions_FINREP.docx/26727402-6339-4c33-bb5a-d8e659c27371` for the instructions on FINREP.

of leverage exposure, whereas other jurisdictions have not.[34] The main results of the paper are obtained under the international standard, while we present the results of adopting the UK standard in Section 4.4.

Liquidity Coverage Ratio. The LCR haircuts are based on the regulatory reporting data in COREP on high quality liquid assets for the purpose of monitoring the liquidity coverage requirement.[35] High quality liquid assets can be divided into three main buckets:

- Level 1 assets, with 0 haircut (with the exception of covered bonds whose market value shall be subject to a haircut of at least 7%). Assets in this category include withdrawable central bank reserves, central government and central bank assets.
- Level 2A assets, with haircut of at least 15%. Assets in this category include regional government assets, high quality covered bonds and corporate debt securities. Assets in this category should exhibit a maximum decline of price over a 30-day period during a relevant period of significant liquidity stress not exceeding 10%. In the case of corporate debt securities, they should also have a long-term credit rating of at least AA-. This implies that, based on the standardised approach they should receive risk weights of 20% for credit risk.[36]
- Level 2B assets, with haircuts between 25% and 50%. Assets in this category include asset-backed securities, corporate debt securities, high quality covered bonds and shares. Assets in this category should exhibit a maximum decline of price over a 30-day period during a relevant period of significant liquidity stress not exceeding 20%. In the case of corporate debt securities, they should also have a long-term credit rating of between A+ and BBB-. This implies that, based on the standardised approach they should receive a risk-weights between 50% and 100% for credit risk. For high quality covered bonds however we know from the LCR classification that the associated risk-weight is 35%.

In total the HQLA contains 54 asset classes, which can be either Level 1, 2A or 2B. We exclude the classes for which all our banks held a negligible market amount.

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[34] Specifically, the UK leverage ratio framework excludes from the calculation of the total exposure measure those assets constituting claims on central banks, where they are matched by deposits accepted by the firm that are denominated in the same currency and of identical or longer maturity; and require a minimum leverage ratio of 3.25%. Central bank claims for these purposes include reserves held by a firm at the central bank, banknotes and coins constituting legal currency in the jurisdiction of the central bank, and assets representing debt claims on the central bank with a maturity of no longer than three months. See Bank of England (2017b) for more details on the UK leverage ratio framework.

[35] See `https://www.eba.europa.eu/documents/10180/359626/Annex+XIII_Instructions_Liquid+assets.docx/38a7b938-9737-4e61-a4be-456910024fb6` for details.

[36] See `http://eur-lex.europa.eu/LexUriServ/LexUriServ.do?uri=OJ:L:2013:321:0006:0342:EN:PDF` and `https://www.eba.europa.eu/documents/10180/16166/4+Ausust+2006_Mapping.pdf`.

This leaves us with 21 asset classes. We then aggregate these classes based on their haircut and balance sheet classification.

Assets also play a role in the LCR through the liquidity inflows in the ratio's denominator. Inflows are defined as contractual cash inflows from outstanding exposures, capped at 75% of total expected cash outflows. This implies that banks must hold at least a stock of HQLA equal to 25% of the total cash outflows. Inflows can come from various sources, such as derivatives or undrawn credit or liquidity facilities.[37] However, for the purposes of the model we need to measure only those inflows coming from tradable assets and collateralized lending. This is because if a bank sells those assets we need to cancel the associated inflows. The remaining inflows coming from different sources are kept as fixed. The reporting template for inflows in COREP categorises collateralised flows by quality of underlying asset or HQLA eligibility. We use this information to match the inflows with the respective HQLA level.

Finally, we use the liquidity outflows data reported in COREP to characterise banks' liabilities. Outflows are reported as the outstanding balances of various types of liabilities (and off-balance sheet commitments), which are then multiplied by the appropriate run-off or draw-down rates for the purposes of the LCR. From the 109 liabilities categories reported in the outflows data, we exclude all the committed facilities and additional outflows from derivatives and other items, leaving us with 44 categories for our model, mostly from unsecured transactions and deposits. We group together liabilities with the same outflow rate. Any residual liabilities (excluding capital) receive a run-off rate of zero.

Risk-weighted capital ratio. Our model requires us to assign risk weights to the different asset classes to compute changes to risk-weighted assets and the capital ratio following sales. The decomposition of the total risk-weighted assets between credit and trading risk has been extracted from the 2016 annual reports and accounts. We ignore counterparty credit risk and operational risk, as they are not relevant for our purposes.

As the regulatory reporting files do not include information on the risk weights or credit ratings of individual securities holdings, we use LCR information on the assets and credit risk steps as set out in the Standardised Approach to Credit Risk[38] as proxies. Moreover, the Basel approach to capital requirements for market risk is based on computing expected shortfall fpr the entire trading book. The computation of expected shortfall relies heavily on the sensitivities of the underlying positions.[39] Since the sensitivities of the positions, as well as potential hedges via derivatives are

[37] We refer to `https://www.eba.europa.eu/documents/10180/930269/Annex+XXV+-+LCR+Instructions+on+inflows.pdf` for a complete list of inflows categories used in the LCR.

[38] See `http://eur-lex.europa.eu/LexUriServ/LexUriServ.do?uri=OJ:L:2013:321:0006:0342:EN:PDF`

[39] See `https://www.bis.org/bcbs/publ/d352.pdf` and `https://www.bis.org/bcbs/publ/d424.pdf` for the Basel III revisions published in 1 December 2017.

not available to us, we apply the standardised approach for credit risk to the portion of market risk that relates to securities (as opposed to derivatives) as a proxy.

In a final step, we scale the risk weights obtained by this procedure such that the asset values in the trading book multiplied by the risk weights are equal to the market risk risk-weighted assets provided in banks' annual reports that relate to securities (as opposed to derivatives). We assume that market risk risk-weighted assets are split proportionally according to the market value of the derivatives book and the securities holdings. As assets on the banking book (available for sale, and held at fair value) count towards credit risk, as opposed to market risk, we perform the same procedure to allocate a portion of risk-weighted assets in credit risk to the securities holdings.

Summary of LCR and RWA mapping. Cash, and low-risk government securities receive an LCR haircut of 0, and we match those exposures carrying a risk weight of 10% with those having a 15 % LCR-haircut. Debt issued by credit institutions or financial corporations that have a 0-haircut LCR categories are also assigned a risk weight of 0. Lower rated debt receives LCR haircuts of 15%, 25%, 50% or 100%. We match these categories with risk weights of 20%, 35 %, 50% and 100 % respectively. For debt issued by non-financial corporations, we have just two classes with a risk weight (and LCR haircut) of either 0% or 100 %. Finally, equities receive a haircut of 50% or 100% for the LCR. We map the 100% haircut equities to the standard risk weight of 250% for equities, and those receiving a 0% LCR haircut to those with a 100% risk weight. Table 6 summarizes the LCR haircuts and risk weights of the different asset classes.

Table 6: Marketable asset categories and regulatory weights

Asset	Exposure	LCR haircut	Risk weight
	Govts and CBs	0	0
		15	20
Debt	Banks and other financials	0	0
		15	20
		25	35
		50	50
		100	100
	Non-financials	0	0
		100	100
Equity		50	100
		100	250

7.2.2 Market depths

The definition and calibration of assets' market depths is a crucial part of the model, as it determines the price impact of assets sales. There is a vast academic literature on how to measure market liquidity, but no consensus on a common approach, in particular for bonds. Furthermore for the purposes of our model, we need a cost per dollar-volume liquidity proxy, in order to quantify price impacts, while many liquidity measures only provide a relative ranking of assets in terms of their liquidity.

We follow Cont and Schaanning (2017) and define an asset's market depth as the ratio between its average trading volume (ADV_k) and the daily volatility σ_k. We use this measure because it can be easily calibrated using publicly available data.

There are finer liquidity measures, but they require detailed microstructure data that are not available for most markets. Some other alternative measures are transaction-by-transaction measures (Hasbrouck, 1991), effective spreads (Hong and Warga, 2000), implementation shortfall (Perold, 1988) and liquidity measures based on high frequency data (Goyenko et al., 2009).

Calibration of market depth when trading volumes are unavailable. For corporate bonds there is no publicly available information on trading volumes that covers all markets. We therefore estimate the average trading volumes using the outstanding amounts in each market.[40] We assume that for all countries the turnover ratio of corporate bonds is proportional to the turnover ratio of government bonds:

$$\frac{ADV_c}{OA_c} = \kappa \frac{ADV_g}{OA_g}$$

where $\kappa = 0.13$ is estimated using US data on turnover ratios.[41] It follows that

$$ADV_c = \kappa ADV_g \times \frac{OA_c}{OA_g}$$

We estimate the daily volatility using prices from S&P bond indices.

7.2.3 Government bonds' market depth data sources

Belgium: Belgian Debt Agency; S&P Belgium government bonds
Canada: IIROC; S&P Canada government bonds.

[40]Data on debt securities outstanding by country are available at `http://stats.bis.org/statx/srs/table/c1`.

[41]The ratio for the US is calculated using volume data from SIFMA. Bond turnover data across markets in Asia, except from Japan, are in line with this estimate. See `https://asianbondsonline.adb.org/regional/data/bondmarket.php?code=Bond_turn_ratio`.

China: AsianBondsOnline; S&P China government bonds.
Finland: AFME; S&P Finland government bonds.
France: French Treasury Agency; S&P France government bonds.
Germany: German Finance Agency; S&P Germany government bonds.
Hong Kong: Hong Kong Government Bond Programme.
Japan: Japan Securities Dealers Association; S&P Japan government bonds.
Korea: AsianBondsOnline; S&P Korea government bonds.
Netherlands: AFME; S&P Netherlands government bonds.
Singapore: Singapore Government Securities; S&P Singapore government bonds.
Spain: Tesoro; S&P Spain government bonds.
Sweden: Riksgalden; Riksbank; S&P Sweden government bonds.
United Kingdom: Debt Management Office; S&P UK government bonds.
United States: SIFMA; S&P US government bonds.

7.2.4 Corporate bonds' market depth data sources

Canada: S&P Canada corporate bonds.
China: S&P China corporate bonds.
Europe: S&P Europe corporate bonds.
Hong Kong: S&P Hong Kong corporate bonds.
Japan: S&P Japan corporate bonds.
Korea: S&P Korea corporate bonds.
Singapore:S&P Singapore corporate bonds.
United Kingdom: S&P UK corporate bonds.
United States: S&P US corporate bonds.

7.2.5 Equities' market depth data sources

Canada: World Bank Canada equities; S&P Canada equities.
Europe: CBOE Europe equities; S&P Europe equities.
Hong Kong: Hong Kong Exchanges; S&P Hong Kong equities.
Japan: Japan Exchange Group; S&P Japan equities.
Korea: World Bank Korea equities; S&P Korea equities.
Singapore: World Bank Singapore equities; S&P Singapore equities.
United States: CBOE US equities; S&P US equities.

7.3 Analytical and numerical solutions

As discussed in Section 2.5, it is possible to find an analytical solution to the problem (7) if we linearise the banks' constraints and they face only one constraint at the time. In this case the problem can be formulated as a continuous Knapsack problem, which has a well known explicit solution.

However these assumptions might not always hold and the optimal solution can differ from the analytical one. Figure 19 plots the bank-level fire-sale losses obtained when solving the original problem numerically against the losses obtained from the analytic solutions in Section 2.5. The numerical solutions are at least as good as those obtained from the analytical approach. Note that when banks face a risk-weighted constraint the solutions coincide, suggesting that the non-linearity in this case is not as pronounced as in the LCR case for instance. Overall, the sequential order given by the Knapsack solution appears to describe banks' optimal strategies well.

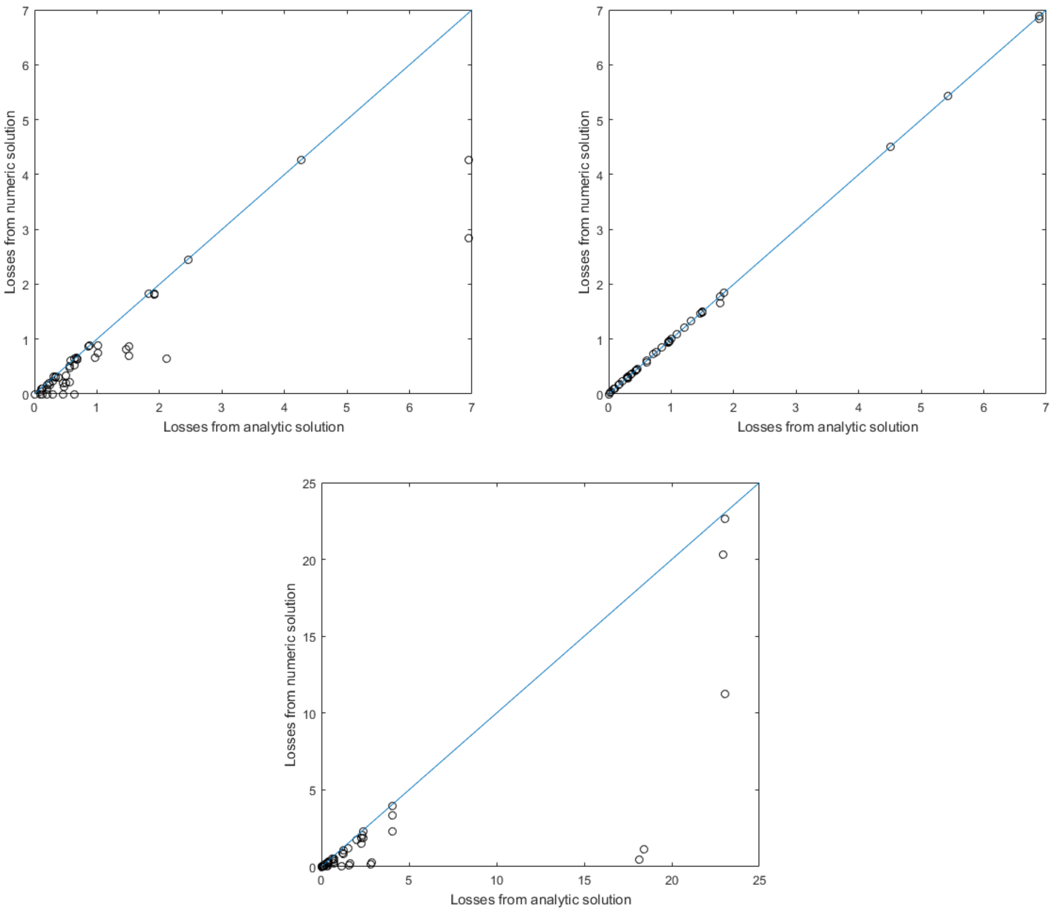

Figure 19: Bank-level fire-sale losses using the numerical solution of the original problem (vertical axis) vs the analytical solution of the linearised problem (horizontal axis). Leverage (top left), risk-weighted capital (top right) and LC (bottom).